DOCUMENT RESUME

ED 081 380 HE 004 563

| | |
|---|---|
| AUTHOR | Wagner, W. Gary; Weathersby, George B. |
| TITLE | Optimality in College Planning: A Control Theoretic Approach. |
| INSTITUTION | California Univ., Berkeley. Ford Foundation Program for Research in Univ. Administration. |
| SPONS AGENCY | Ford Foundation, New York, N.Y. |
| REPORT NO | Pap-P-22 |
| PUB DATE | Dec 71 |
| NOTE | 72p. |
| AVAILABLE FROM | Ford Foundation, 2288 Fulton Street, Berkeley, California 94720 |
| EDRS PRICE | MF-$0.65 HC-$3.29 |
| DESCRIPTORS | *College Planning; *Decision Making; *Educational Planning; Educational Policy; *Higher Education; Policy Formation; *Program Planning; Resource Allocations |

ABSTRACT

In this paper the authors argue that the decision structures of educational institutions are multi-level, multi-decision-maker hierarchies which can be described and analyzed in decision theoretic terms and that these multi-levels, multi-decision-maker hierarchies can be reduced to equivalent one-level, one-decision-maker formulations, which can be solved either analytically or numerically by the techniques presented. Illustrative examples are given which identify and then solve for a set of optional resource allocation and policy decisions. The computer program used for the problem and the input data specifications are included in an appendix. A 24-item bibliography is included. (Author)

FILMED FROM BEST AVAILABLE COPY

ED 081380

# FORD FOUNDATION PROGRAM FOR RESEARCH IN UNIVERSITY ADMINISTRATION

Office of the Vice President—Planning
University of California

HE 004 563

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

FORD GRANT NO. 680-0267A

RESEARCH DIRECTORATE


| | |
|---|---|
| Charles J. Hitch | President, University of California |
| Frederick E. Balderston | Professor of Business Administration<br>Chairman, Center for Research in Management Science<br>University of California, Berkeley<br>Academic Assistant to the President |
| George B. Weathersby | Associate Director, Office of Analytical Studies<br>University of California |

OFFICE ADDRESS

2288 Fulton Street
Berkeley, California 94720
(415) 642-5490


(List of Available Publications on Inside Back Cover)

ED 081380

# OPTIMALITY IN COLLEGE PLANNING:
# A CONTROL THEORETIC APPROACH

W. Gary Wagner

George B. Weathersby

Paper P-22

December, 1971

TABLE OF CONTENTS


| | Page |
|---|---|
| PREFACE | ii |
| LIST OF TABLES | iii |
| LIST OF FIGURES | iv |
| INTRODUCTION | 1 |
| INSTITUTIONAL DECISION MAKING | 4 |
| Analytical Description of Institutional Decision Making | 11 |
| The President's Model: Single Decision Maker Paradigm | 15 |
| SMALL CAMPUS PLANNING MODEL | 19 |
| SYSTEM DYNAMICS OF SCPM | 20 |
| Faculty | 20 |
| Students | 21 |
| Space | 22 |
| Money | 22 |
| CRITERION FUNCTION | 25 |
| Student/Faculty Ratio | 25 |
| Faculty Mix | 26 |
| Space | 27 |
| Student Mix | 27 |
| Monetary Balance | 28 |
| Terminal Conditions | 29 |
| SUMMARY OF RESULTS | 30 |
| Comparison of Optimal versus Sub-Optimal Results | 34 |
| Comparison of Results from Various Policy Alternatives | 40 |
| Conclusion | 46 |
| APPENDIX | 47 |
| Solution Procedure | 47 |
| Computer Program | 48 |
| Data Inputs for Multiple Runs | 60 |
| Form of Input Data | 61 |
| Data Structure for Multiple Runs | 62 |
| BIBLIOGRAPHY | 63 |

## PREFACE

This is one of a continuing series of reports of the Ford Foundation sponsored Research Program in University Administration at the University of California, Berkeley. The guiding purpose of this Program is to undertake quantitative research which will assist university administrators and other individuals seriously concerned with the management of university systems both to understand the basic functions of their complex systems and to utilize effectively the tools of modern management in the allocation of educational resources.

In this paper we argue that the decision structures of educational institutions are multi-level, multi-decision-maker hierarchies which can be described and analyzed in decision theoretic terms. Furthermore, we show that these multi-level, multi-decision-maker hierarchies can be reduced to equivalent one-level, one-decision-maker formulations, which can be solved either analytically or numerically by the techniques presented in this paper. An illustrative example is given which first identifies and then solves for a set of optimal resource allocation and policy decisions. A listing of the computer program used in this problem and the input data specifications are included in the Appendix.

LIST OF TABLES


| | | Page |
|---|---|---|
| Table 1: | Examples of Interrelationships of Institutional Decision Makers | 6 |
| Table 2: | Accounting Summary for SCPM Example | 31 |
| Table 3: | Data Set 1 | 32 |
| Table 4: | Comparison of Control Variables - Data Set 1 - Unconstrained vs Non-Negative Modes | 36 |
| Table 5: | Comparison of Unrestricted Funds Balances - Unconstrained vs Non-Negative Modes (Data Set 1) | 37 |
| Table 6: | Comparison of Unconstrained (A) vs Non-Negative (B) Modes - Preference Function Targets (Data Set 1) | 39 |
| Table 7: | Comparison of Data Set 1 and Data Set 2 - Enrollments and Costs | 42 |
| Table 8: | Comparison of Data Set 1 and Data Set 3 - Enrollments and Costs | 44 |
| Table 9: | Comparison of Data Set 1, Data Set 2, Data Set 3, and Data Set 4 - Enrollments and Marginal Costs | 45 |

LIST OF FIGURES


| | | Page |
|---|---|---|
| Figure 1: | Operations Cycle of an Institution: Decision - Execution - Evaluation - Decision . . . . . | 8 |
| Figure 2: | Interrelationship of Decisions and Values in a Hierarchical Decision Structure . . . . . | 10 |
| Figure 3: | Array of Decision Makers in Three Level Hierarchy. . . . | 14 |
| Figure 4: | Comparison of Control Variables - Unconstrained vs Non-Negative Modes (Data Set 1) . . . . . | 38 |

# INTRODUCTION

Institutions of higher education currently face a number of major policy choices which will largely determine their character for the next twenty-five years. The tremendous expansion of American higher education in the last twenty-five years was driven by burgeoning enrollment growth and by massive federal commitment to doctoral production in the sciences and technologies. Both of these forces are abating rapidly. Nationally, enrollments in higher education are forecasted to peak in 1980, then decline until the late 1980's and not approach the 1980 level until after 1995. Many schools are now experiencing enrollment levels below their previous expectations. This is not a short-run phenomenon; rather, current enrollment shortfalls are harbingers of the next twenty-five years. Colleges and universities must learn to survive and to prosper with a decreasing demand for their services.

It is far less likely that in the future the federal government will rescue the expectations of higher education as they have done in the post-Sputnik era. The United States will probably have a surplus of highly trained scientific and technical manpower for at least the next decade without major additional federal expenditures [Brode (1971)]. The reduced rate of undergraduate enrollment expansion will drastically reduce the number of new teachers needed in colleges and universities, thereby reducing the future demand for additional Ph.D.'s and, therefore, the need for large doctoral programs. Furthermore, the federal priorities have shifted from scientific manpower to equality of student access and the quality of educational experience. Both of these major federal

objectives impact undergraduate education far more than graduate programs and they move counter to many institutions' prestige and elitist orientations.

While the demands for educational services by students and governments will probably be decreasing in the coming decades (first in rate of growth and then in absolute number), the costs of educational institutions continue to rise. As Cheit [1970] has pointed out, a significant number of America's colleges and universities are headed for financial difficulties and current institutional rigidities preclude those cost adjustments necessary to maintain fiscal viability.[1] Furthermore, the technology of education has changed very little in the last three or four decades; indeed, some would argue that educational technology has changed very little since Socrates. In essence, there has been no observable productivity increase in American post-secondary education in the last four decades [O'Neill (1971)].

If it were not so painful, we might examine with considerable intellectual interest the experience of public and private eleemosynary institutions beset by diminishing demands for services and rising costs, increasing institutional rigidities and no productivity increases.[2] Unpleasant as it may be, educational administrators are having to ask the

---

[1]The "varieties of the financial crisis" are explored by Balderston [1971] in a recent paper prepared for the American Council on Education.

[2]This description closely resembles the experience of the American railroads. One inciteful observation on the decline of the railroad companies was that unfortunately railroad managers viewed their industry as "railroads" versus "transportation." At the time of burgeoning new modes of transportation, the railroad companies were in an excellent position to diversify and expand--but that was neither their tradition nor their self-concept. Are our schools in the "formal instruction" industry or the "education" industry?

tough questions: What are our objectives? How would we know if we achieved them? How can we reallocate resources to be more productive? What activities are really essential for an educational institution? Who should make these decisions; and many others?

The purpose of this paper is to look at institutional resource management decisions in the context of institutional goals and objectives. After describing one view of institutional decision making, we present a simple yet comprehensive mathematical model which explicates the interrelationships of major institutional variables. Sample data are then used to derive resource allocations which would be optimal for the institution. The use of this model in educational policy analysis is then discussed before presenting our conclusions.

## INSTITUTIONAL DECISION MAKING

The decision making structures of educational institutions are as diverse as the institutions themselves. Some colleges and universities are highly authoritarian while others are highly egalitarian; some institutions are ruled by presidents and others by committees. Some educational systems have many layers of administrative superstructure while others do not. There is often little resemblance between the organizational structure and the decision or power structure of an institution. Often individuals with no delegated authority have great influence on decisions.

While these complex interactions have been analyzed from many perspectives,[3] we have chosen to analyze institutional decision making from a decision theoretic basis. Initially we distinguish between the values used in arriving at policy decisions and the authority structure in which the decisions are made. Focusing first upon the structure of decisions, we observe that the decision structures of most educational institutions are hierarchical, with students, individual faculty members, department chairmen, deans, provosts, and presidents playing different, but important, decision making roles.

These roles are distinguished primarily by the variables each level can control. For example, students decide which of the available courses they will take; faculty decide how to allocate their time between formal

---

[3] Wildavsky [1964] looks at the resource allocation process in government from a political theory perspective; Downs [1967] and Braybrooke and Lindblom [1963] view bureaucratic decision making as a behavioral and organizational process; Glenny [1969] and Palola [1970] approach educational decision making from the perspective of governance.

instruction, preparation, informal meetings with students, research, committees, community service, professional advancement, and leisure; department chairmen decide, with consultation to be sure, the course and committee assignments of faculty, the allocation of support services, recommendations on salaries and promotions, and curriculum proposals; deans allocate new faculty positions to departments, increasingly will reallocate faculty positions between departments, determine salaries and promotions, establish departmental budgets, endorse curriculum changes, and approve research programs; provosts or presidents in turn allocate faculty positions and budgets between schools, review or approve salaries and promotions, recruit deans, approve curricula and academic programs. Table 1 summarizes some of these distinctions.

Another characteristic of the hierarchical structure of educational decision making is the direct interrelationships of the various decision making levels. As illustrated in Table 1, the control variables at one level often become constraints at the next lower level. For example, the president can allocate faculty positions to the various schools in his institution to the limit of his budget. In turn, deans can allocate faculty to departments up to the limit permitted by the president's budget. What was initially a decision to the president later becomes a constraint to the dean.

Another component of our analysis of institutional decision making is the distinction between the implementation structure and the decision structure. The implementation structure is usually reflected in the institution's organization chart; it is the array of deans, department chairmen, accounting officers, purchasing agents, budget officers, admissions officers, registrars, librarians, and all the other functional

TABLE 1

Examples of Interrelationships of Institutional Decision Makers

| Decision Makers | Control Variables | Constraints |
|---|---|---|
| President/Provost | Budgets | Income or Appropriations |
| | Faculty Positions | |
| | Program Approval | |
| Dean | Departmental Budgets | President's Budget |
| | Faculty Positions | President's Budget |
| | Program Approval | President's Approval |
| Department Chairmen | Faculty Assignments | Dean's Budget |
| | Support Services | Dean's Budget |
| | Salaries and Promotions | Dean's Budget |
| | Curriculum | Dean's Approval |

specialists who keep an institution running effectively. On the other hand, the decision structure is rarely reflected in a school's organization chart. At issue here is who is responsible for what decisions and how are the recommendations for these decisions made.

The operations cycle of an institution is illustrated in Figure 1. Once made, a decision is communicated to the implementation structure where functional specialists establish the operating policies and procedures which actually move the organization in the desired direction. These implementation managers need operating data for their effective functioning. For example, the accounting officer needs payroll information to process checks and charge the appropriate accounts. On the other side of this circle, those who are charged with recommending decisions need institutional data to evaluate past decisions and as input to future decisions. In addition, external data on student demand, manpower supply and demand, community needs, attractive research areas and a variety of other issues are needed for decision recommendations. The process of synthesizing these data and institutional objectives into a coherent, consistent strategy for future action is nebulous if not nonexistent at most institutions. Yet, if educational leaders are going to be able to deal effectively with the serious social and economic challenges confronting their institutions, much more attention will have to be devoted to their decision structures. The approach and the mathematical model presented in this paper is one small step in this direction.

In addition to the process of decision are the values upon which the decision is based. One of the functions of each decision maker is to choose the values appropriate for the decisions at hand. This is another way of raising the question of governance: Who will

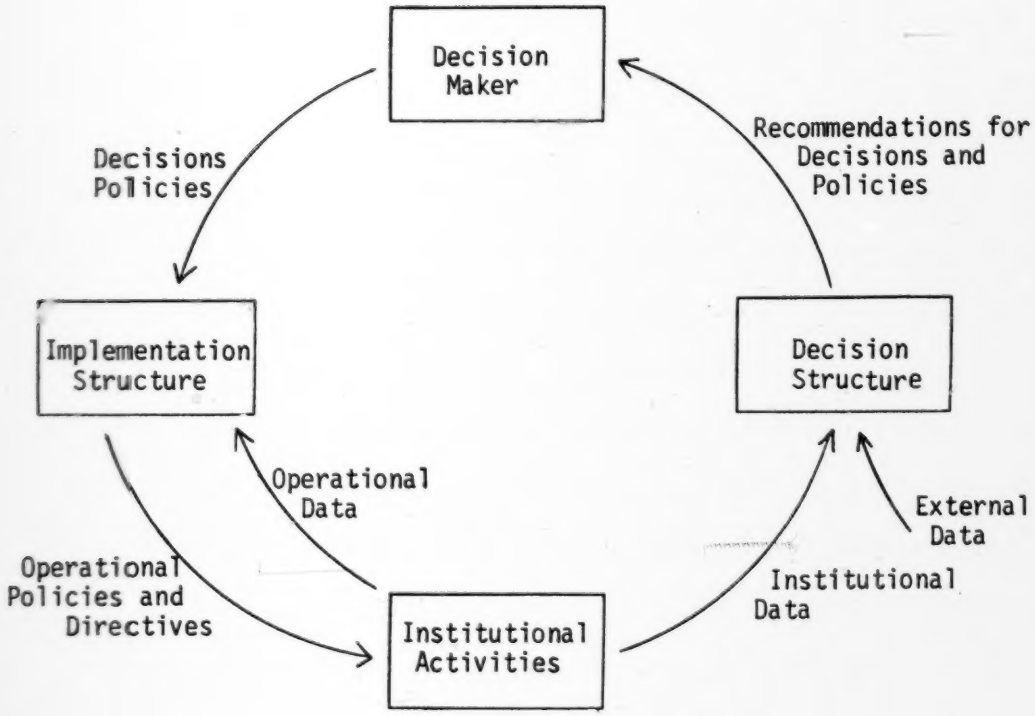


FIGURE 1

Operations Cycle of an Institution:
Decision - Execution - Evaluation - Decision

decide and whose values will he use when he decides? Furthermore, what attributes of the educational system are important to the decision maker, what does he consider to be the outputs of his system?[4] How important are more undergraduates versus more graduate students, more researchers versus more instructors, more computing power versus more library services, more faculty versus more facilities, and a thousand-and-one other possible tradeoffs? Taken together, all of these choices and tradeoffs comprise a decision maker's value system.

These value systems also serve to connect the hierarchical decision systems which were discussed earlier. In many cases, the president of an institution is deeply concerned about the classroom environment and the interaction of students and faculty, even though he cannot directly control any of the operative variables. However, the president often makes his budgetary and faculty allocations with their educational consequences in mind and adjusts his allocations to correspond to his assessment of the educational use to which these resources are put. In other words, the consequences of decisions at a lower level are important to decision makers at higher levels.

There is a circular flow of information in a hierarchical decision system: decisions are passed downward and value signals are passed upward. These in turn affect the decisions which are passed downward in a subsequent cycle, as shown in Figure 2. It is this two-way flow of information that makes delegated authority operational and renders a decentralized or hierarchical system controllable; this notion of decentralized control will be explored in more detail shortly.

---

[4] The outputs of higher education have received increasing attention in the last few years; see Lawrence, Weathersby and Patterson [1970], Breneman and Weathersby [1970] and Huff [1971].

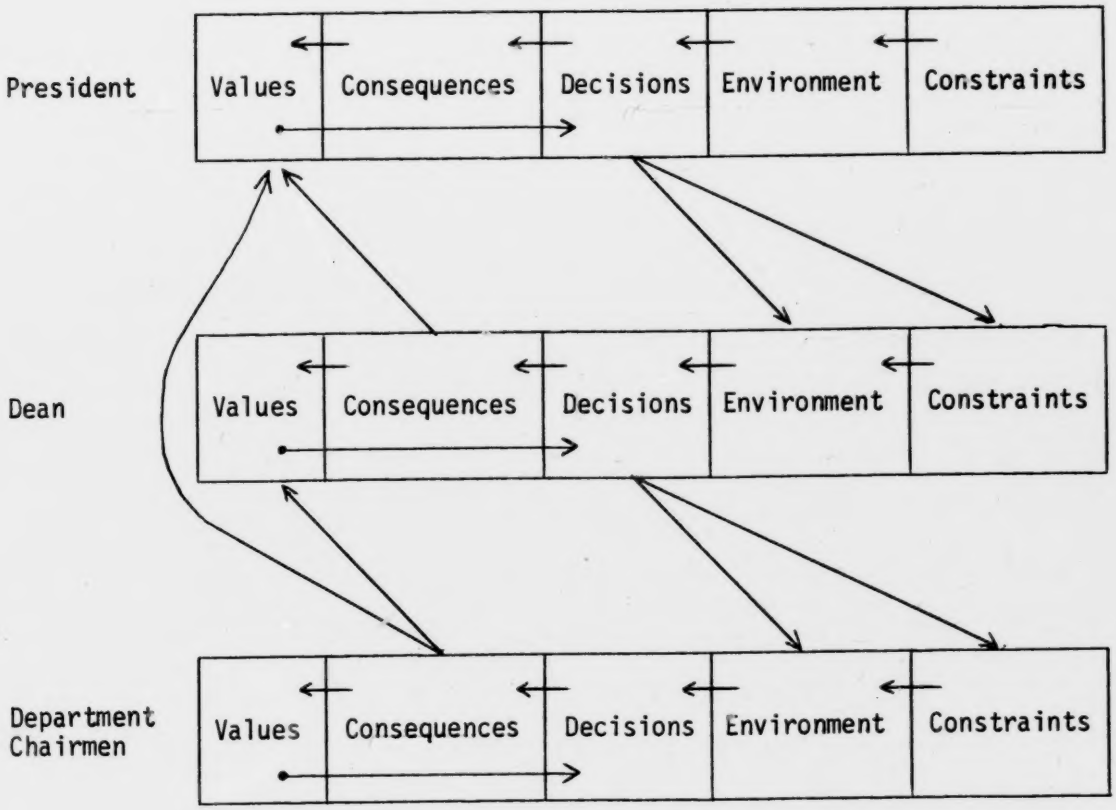


FIGURE 2

Interrelationship of Decisions and Values
in a Hierarchical Decision Structure

## Analytical Description of Institutional Decision Making

This conceptual analysis of educational decision making can be made more precise by describing the decision interrelationships in mathematical terms. To begin with, we need some definitions:[5]

$u_i(t)$ = the vector of decision variables available to decision maker (DM) $i$ in period $t$ ;

$z_i(t)$ = the vector of predetermined variables impinging upon the system relevant to DM $i$ in period $t$ ;

$x_i(t)$ = the vector of consequences (or state variables) in period $t$ resulting from the decisions of DM $i$ and relevant exogenous influences.

The relationship of consequences to decisions (or output to input) is often called the production function:

$$x_i(t+1) = f_i(x_i(t), u_i(t), z_i(t), t). \qquad (1)$$

Finally, the value to $DM_i$ of making a decision $u_i(t)$ when confronted with the predetermined variables $z_i(t)$ is written

$$V_i(x_i(t), u_i(t), z_i(t), t). \qquad (2)$$

Expressions (1) and (2) describe the horizontal flows shown in Figure 2 at each decision making level.

The decision problem faced by each administrator is to maximize his own values subject to his constraints of authority and resources and subject to the responsiveness of his system to the application of policy or resource decisions. Furthermore, a decision maker often looks several

---

[5] For a complete exposition of this approach and the motivation for these definitions, see Weathersby [1969a, 1969b].

years in advance and wants to maximize his values over a planning horizon of N periods. We may write this decision problem as:[6]

$$\max_{u_i(o),\ldots,u_i(N-1)} \left\{ J = \sum_{t=o}^{N} V_i\left[x_i(t),\ u_i(t),\ z_i(t),\ t\right] \right\} \qquad (3)$$

subject to:

$$x_i(t+1) = f_i\left[x_i(t),\ u_i(t),\ z_i(t),\ t\right] \quad t=o,1\ldots,n-1 \qquad (1)$$

$$x_i(o) \text{ known}$$

and

$$C\begin{bmatrix} x_i(t) \\ u_i(t) \end{bmatrix} \leq b_i(t) \qquad t=o,1\ldots,n-1\ . \qquad (4)$$

In this formulation, C is the constraint function and $b_i(t)$ are the resource and other constraints relevant to $DM_i$ in period t .

In general, there is a solution $u_i^*(t)$, t=0,1,...,n-1 which maximizes the overall value function, J , provided the necessary and sufficient conditions are satisfied.[7] Furthermore, an optimal solution is in general a function of all preceding variables.[8]

$$u_i^*(t) = g_i\left[u_i^*(o),\ldots,u_i^*(t-1),\ x_i(o),\ldots,x_i(t),\ z_i(o),\ldots x_i(t),\right.$$
$$\left. b_i(o),\ldots b_i(t),\ t\right]\ . \qquad (5)$$

---

[6]The general form of the N period value function is $V_i\left[x_i(o),\ x_i(1),\ldots,x_i(N),\ u_i(o),\ u_i(1),\ldots,u_i(N-1),\ z_i(o),\ldots,z_i(N-1)\right]$ which can be separated into $\sum_{t=o}^{N-1} V_i\left[x_i(t),\ u_i(t),\ z_i(t),\ t\right] + V_i\left[x_i(N),\ N\right]$ by the assumption of weak separability (Weathersby [1969a]). Notice that present value discounting is a special case of the time function of V(x,u,z,t).

[7]The solution procedure will be described in more detail later and the algorithm used in this study is described in the Appendix.

[8]See Aoki [1969] for a discussion of general recursive solution.

The consequences which result from an optimal decision sequence can be calculated from equation (1);

$$x_i^*(t+1) = f_i\left[x_i(t),\ u_i^*(t),\ z_i(t),\ t\right]$$

$$= h_i^*\left[x_i(o),\ldots,x_i(t),\ z_i(o),\ldots,z_i(t),\ b_i(o),\ldots,b_i(t),t\right] \tag{6}$$

after equation (5) is substituted for $u_i^*(t)$. In other words, when the parameters of the decision problem are known, i.e., equations (1), (3), (4), and (5), one can replace the decision problem by equation (6) which describes the consequences of an optimally controlled system.

In a strict hierarchical decision structure, there is one such decision problem for each decision maker. In the three level hierarchy shown in Figure 3, $n$ deans report to the president and $m_i$ department chairmen report to the $i^{th}$ dean. The interrelationships are:

1. At levels 1 and 2, the predetermined variables $z(t)$ and resource constraints $b(t)$ can be controlled or influenced at the next higher level. For example,

$$x_{1,1}(t+1) = f_{1,1}\left[x_{1,1}(t),\ u_{1,1}(t),\ z_{1,1}(t),\ u_{2,1}(t),\ z_{2,1}(t),\ u_{3,1}(t),\ z_{3,1}(t)\right].$$

2. The values associated with the consequences of decisions by decision-makers at levels 2 and 3 can include the decisions and consequences of lower level decision-makers. For example:

$$V_{3,1} = V_{3,1}\left[x_{3,1}(t),\ u_{3,1}(t),\ z_{3,1}(t),\ x_{2,1}(t),\ x_{2,2}(t),\ldots,x_{2,n}(t),\ u_{2,1}(t),\ldots,u_{2,n}(t),\ t\right] .$$

The first interrelationship describes the downward flow of decisions either directly or indirectly while the second form of interrelationship describes the upward flow of accountability or value. Both of these interrelationships

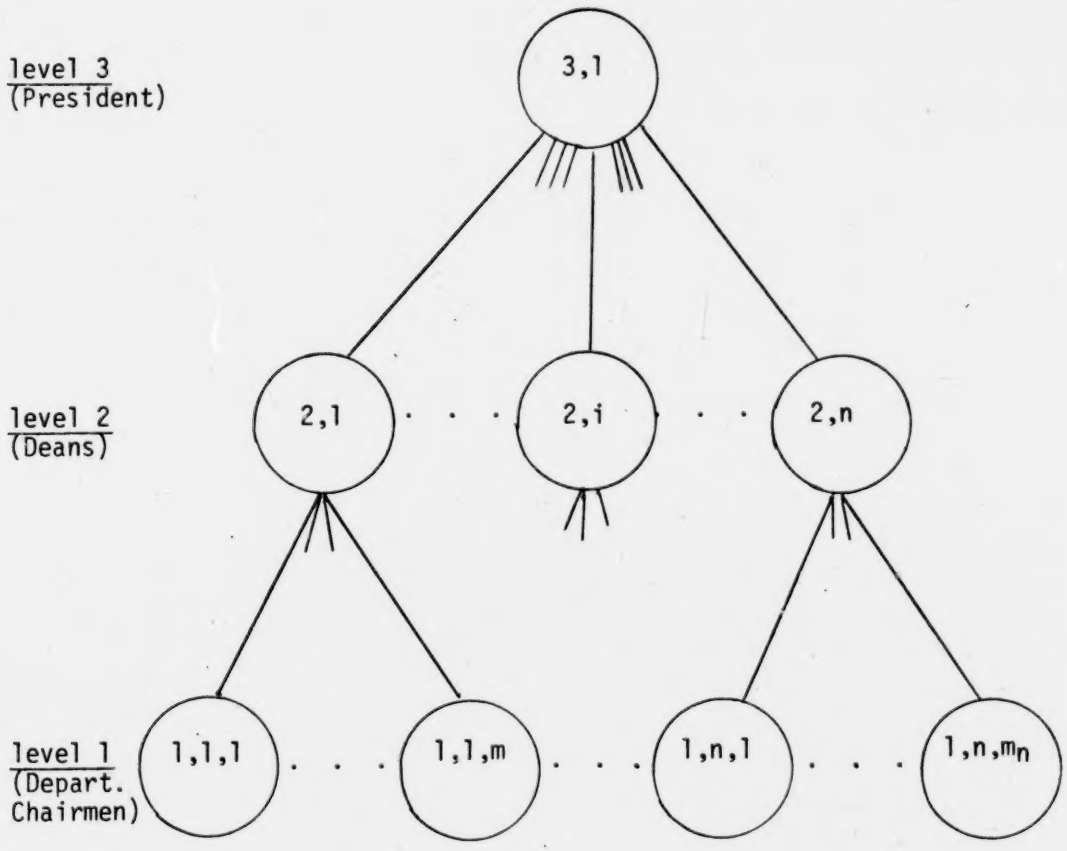


FIGURE 3

Array of Decision Makers in Three Level Hierarchy

---

are necessary for a controllable hierarchical system; however, they are not sufficient conditions for total systems controllability.

The basic strategy of solution of a strict hierarchical decision structure (i.e., interrelationships only between adjoining levels) is to reduce the structure down to a one decision maker problem by folding up from the bottom.[9] This approach would replace each decision making node at level 1 in Figure 3 by his corresponding optimal decision function $g^*_{1,i,j}$ and the corresponding optimal production function $h^*_{1,i,j}$. For each level 2 decision maker, the $m_i$ departmental chairmen's decision

---

[9] For a complete presentation of this approach, see Weathersby [1969b].

problems are replaced by $2m_i$ vector equations for the $g^*$ and $h^*$ functions. These $2m_i$ vector equations then are effectively production functions to the $i^{th}$ dean which augment his own production function. Now we can fold the dean's decision problems up to the presidents' level by the same technique. Thus, we can collapse a multi-layer, multi-decision maker hierarchy to an equivalent one-decision-maker problem. Correspondingly, if we can solve the one-decision-maker decision problem, we can conceptually solve the multi-layer, multi-decision maker problem. Therefore, the remainder of this study will focus on the single decision maker problem.

## The President's Model: Single Decision Maker Paradigm

The basic decision problem of the president is to maximize the achievement of his own values, or the values he chooses to operate with as president, subject to resource limitations and the responsiveness of his institution. The formal statement of this problem was given previously in equations (3), (1), and (4). The three major components of the problem are: (1) the president's value function; (2) the institutional response or production function; and (3) the resource, legal and other constraints.

One obvious difficulty with the decision theoretic formulation is that generally presidents, and other administrators, cannot articulate their value function. Most of us are not trained to think in terms of multi-attributed utility functions and, therefore, any approach which requires a mathematical description of a decision maker's value function is destined to grave difficulty if not failure.

There have been two major techniques for circumventing the assessment difficulties associated with a full description of the value function.

Geoffrion and Dyer [1970, 1971] have shown that one need only assess the local gradient of the value function at the current operating point. In their work, they ask a dean or department chairman to select one of his variables as a numeraire and then assess the pairwise tradeoffs of all the other policy relevant variables with respect to the chosen numeraire. This is the local gradient which shows an improving direction along which the decision maker selects a new and improved operating point. At this new point, however, the local gradient must be assessed again because it is generally different at every point on the utility surface. In other words, the Geoffrion and Dyer approach replaced a global assessment of the multidimensional value surface with a series of local assessments of the tangent plane, which is a much easier task. This requires interaction between the decision maker and the mathematical programming algorithm because the path along the value surface is unpredictable a priori.

A second approach to the reduction of the dimensionality of the value assessments is to express the decision-maker's objectives in terms of targets. This is the approach used in the study reported in this paper. For simplicity of exposition, consider a president's value function that is defined over only the consequences of state vector $x$, i.e., $V(x)$, and that the president wants to achieve a most desirable level of $x$, say $x^*$. In other words, the president believes that the optimal state of his institution would be a student enrollment of 10,000 with 1,000 faculty members, 600 of whom would be tenured, and so forth.

The key to the target approach is that if the institution is initially reasonably near the desired targets, then the general utility maximization problem can be expressed by an approximately equivalent loss minimization problem where the loss function is quadratic, independent of the form of

the utility function as long as the utility function is twice differentiable, i.e., smooth and continuous. Although we may know nothing more about the president's utility function than that it ought to be concave and smooth,[10] we do know that in the neighborhood of his targets his loss function is quadratic to second order.

By choosing targets $x^*$, a decision maker indicates that

$$V(x^*) > V(x) \quad \text{for all} \quad x . \tag{7}$$

For $x$ near $x^*$, we can expand $V(x)$ about $x^*$ by Taylor series

$$V(x) = V(x^*) + \nabla V\big|_{x^*}(x-x^*) + 1/2(x-x^*)^t \nabla^2 V\big|_{x^*}(x-x^*) + \text{Higher Order Terms.} \tag{8}$$

If $V(x^*)$ is a maximum, as indicated by expression (7), then the local gradient must be zero at $x^*$ and the second right hand side term in equation (8) must be zero. Furthermore, the second derivative of $V$ must be negative definitive for $x^*$ to be a strict maximal point. Therefore, the third right hand side term in equation (8) must be negative for all $x$. This argument proves that, to second order,

$$\max_x V(x) \doteq \min_x 1/2\left\{(x-x^*)^t \nabla^2 V\big|_{x^*}(x-x^*)\right\} . \tag{9}$$

One point of indeterminancy remains in equation (9); in general, the matrix of second partial derivatives of $V$ is not known. Two approaches may be used here. One can ask the decision maker to choose a numeraire and assess the relative pairwise comparison losses that he would exper-

[10] We can assess the relative relationship between the first and second derivatives of a decision-maker's utility function by a discussion of his risk aversion (Pratt [1964]).

ience at $x^*$ and use this one set of assessments in place of $\nabla^2 V$. Alternatively, one could recognize that the magnitudes of $\nabla^2 V$ change the relative shape of the quadratic loss structure but not its minimum, which is $x^*$. Near the minimum, the solution to (9) is often insensitive to the global shape of $\nabla^2 V$ and a much simpler procedure is possible: namely choose an arbitrary weighting matrix $K$ in place of $\nabla^2 V$ such that the magnitudes of loss of one unit variation in every dimension are identical. Both of these approaches require minimal assessment.

In summary, we have argued that the decision structures of educational institutions are multi-level, multi-decision-maker hierarchies which can be described and analyzed in decision theoretic terms. Furthermore, we have argued that these multi-level, multi-decision-maker hierarchies can be reduced to equivalent one level, one decision-maker formulations. In turn, these single decision-maker problems can be solved either analytically or numerically by the techniques discussed in this and the following section. We now proceed with the formulation and solution of a specific decision model and discuss its implications.

## SMALL CAMPUS PLANNING MODEL

The concepts of the previous section are illustrated in this section in a specific analytical modeling context. For the purposes of exposition, we have focused on the instructional program of an institution partly because this seems to be an area of great interest to most colleges and partly because instructional activities have far more in common among institutions than the various research and public service programs. The paradigm of this model is the liberal arts undergraduate institution or that component of a major university.

The Small Campus Planning Model (SCPM) is designed to provide a control theoretic solution to the problem of finding an optimal sequence of new student admissions, new faculty hires, and new physical construction over an N-year planning horizon. It assumes that the flows of students, faculty, construction, and money can be characterized by linear dynamic equations and that the campus administrator's preferences for student and faculty mix, for space, and for solvency are sufficiently close to the institution's current experience that actual deviations from targets can be adequately expressed in terms of quadratic penalty functions.

The model is still in the investigatory stages and will undoubtedly undergo further revision before it is considered a finished product. Ultimately it is hoped that SCPM, because of its minimal data requirements and low implementation and calculation costs, may serve as a useful planning device for college administrators who have neither the funds nor the data base to support implementation of other, more complex, models.[11]

[11] For a structural comparison of other recent analytical models for university planning, see Weathersby and Weinstein [1970].

## SYSTEM DYNAMICS OF SCPM

The generalized form of SCPM's dynamics may be characterized by

$$x(t+1) = F_t\, x\,(t) + G_t\, u(t) + H_t\, z(t) \qquad (1)$$

where:

$x(t)$ = n-vector of state variables at time $t$
$u(t)$ = m-vector of control variables at time $t$
$z(t)$ = n-vector of predetermined variables at time $t$
$F_t$ = n x n matrix of transition coefficients for period $t$
$G_t$ = n x m matrix of transition coefficients for period $t$
$H_t$ = n x n matrix of transition coefficients for period $t$ .

It is assumed in SCPM that $F_t = F$ , $G_t = G$ , and $H_t = H$ for all $t$ , i.e., that the transition matrices are not time dependent. This is not necessary for solution, but facilitates estimation and reduces substantially the data requirements. Furthermore, these matrices are not Markovian, i.e., the row sums do not total 1.0, because the absorption states for students, faculty, space and money are excluded. Under the stationarity assumption, equation (1) becomes

$$x(t+1) = F\, x\,(t) + G\, u(t) + H\, z(t) \quad . \qquad (2)$$

The state, control, and predetermined variables as defined in SCPM are divided naturally into four groups: students, faculty, space, and money. We consider each separately for purposes of exposition.

### Faculty

We define the variables $x_1(t)$ to $x_4(t)$ to be the number of full-time equivalent full professors, associate professors, assistant professors

and instructors who were in the institution last year (at time t-1) and who remained in the system at the start of this year (period t). Similarly, we define the variables $u_1(t)$ to $u_4(t)$ to be the number of faculty who are hired at corresponding ranks at the start of period $t$. Then, expressing the matrices $F$ and $G$ by their elements $f_{ij}$ and $g_{ij}$, we have

$$\begin{aligned} x_1(t+1) &= f_{11}x_1(t) + f_{12}x_2(t) + g_{11}u_1(t) + g_{12}u_2(t) \\ &\vdots \\ x_4(t+1) &= f_{44}x_4(t) \qquad\qquad + g_{44}u_4(t) \ . \end{aligned} \tag{3}$$

Here, $f_{ij}$ is the promotion rate of faculty from level $j$ in period $t$ to level $i$ in period $t+1$, $f_{ii}$ is the continuation rate for faculty at the same rank, and $g_{ij}$ is the promotion and continuation rates of new faculty who were hired in period $t$ at level $j$ and who are at level $i$ in period $t+1$.

## Students

We define the variables $x_5(t)$ to $x_8(t)$ to be the number of continuing freshman, sophomore, junior and senior students at the start of period $t$. Similarly, we define the variables $u_5(t)$ to $u_8(t)$ to be the number of students admitted to the corresponding student levels at the start of period $t$. Once again, we may write the scalar equations

$$\begin{aligned} x_5(t+1) &= f_{55}x_5(t) \qquad\qquad + g_{55}u_5(t) \\ &\vdots \\ x_8(t+1) &= f_{87}x_7(t) + f_{88}x_8(t) + g_{87}u_7(t) + g_{88}u_8(t) \ . \end{aligned} \tag{4}$$

The coefficients $f_{ij}$ and $g_{ij}$ have the same advancement and retention interpretations as before. Attrition of faculty and students is accounted for by omission of state variables corresponding to the "out" state. For

manpower planning or other purposes one could define two additional states of successful degree completion and "stopping out." This would provide specific degree output information and render the student system Markovian.

## Space

We assume that physical construction takes an average of four years to complete once it has begun. A conscious simplification at this stage is the assumption of fully interchangeable space types and uses. Equation (5) could be repeated for each space type if the additional detail would be worth the additional cost. SCPM also assumes a constant depreciation rate of $\alpha = [1-f_{12,12}]$ . Accordingly we define

$u_9(t)$ = amount of new construction measured in Assignable Square Feet (ASF) which begins in period t

$x_9(t)$ = ASF begun in period t-1

$x_{10}(t)$ = ASF begun in period t-2

$x_{11}(t)$ = ASF begun in period t-3

$x_{12}(t)$ = ASF which is available and usable at the start of period t .

Thus,

$$\begin{aligned} x_9(t+1) &= u_9(t) && \text{i.e., } g_{9,9} = 1.0 \\ x_{10}(t+1) &= x_9(t) && \text{i.e., } f_{10,9} = 1.0 \\ x_{11}(t+1) &= x_{10}(t) && \text{i.e., } f_{11,10} = 1.0 \\ x_{12}(t+1) &= x_{11}(t) + f_{12,12}\, x_{12}(t) && \text{i.e., } f_{12,11} = 1.0 \ . \end{aligned} \tag{5}$$

## Money

Finally, we assume that there are two kinds of funds which adequately describe the administrator's financial concerns: restricted funds (endowment) and unrestricted funds (operating plus capital funds). Once again,

these fund categories could be expanded if needed. It is further assumed that interest earned on endowment funds may be allocated arbitrarily between funds, but that the income and capital gains use policy is fixed in advance. If $f_{14,j}$ is the value (cost if negative) of one unit of $x_j$ and $g_{14,j}$ is the value (cost) of one unit of $u_j$, and performing all calculations in constant dollars, we can define

$z_{13}(t)$ = restricted gifts in period $t$ (estimated or assumed known)

$z_{14}(t)$ = unrestricted gifts in period $t$ (estimated or assumed known)

and write:

$$x_{13}(t+1) = f_{13,13}x_{13}(t) + h_{13,13}z_{13}(t) \tag{6}$$

$$x_{14}(t+1) = \sum_j f_{14,j}x_j(t) + \sum_j g_{14,j}u_j(t) + h_{14,13}z_{13}(t) + h_{14,14}z_{14}(t) .$$

While equation (6) looks quite complicated, each of its components is very simple and traditional.

$f_{14,1}$ to $f_{14,4}$ = the average faculty salary by rank including direct support costs.

$f_{14,5}$ to $f_{14,8}$ = the net institutional cost per student by level (excluding faculty salaries and direct faculty support costs and including student fees and tuition).

$f_{14,9}$ to $f_{14,12}$ = the average cash flow cost per ASF in each year of new construction.

$f_{14,13}$ = the average rate of return on endowment that is available for operating expenses.

$f_{14,14}$ = the proportion of last year's net cash balance available in the current year (usually 1.0).

The numerical values of these and the other coefficients used in the computational example are given in Table 3. Note that equation (2) is just the aggregate of equations (3) through (6). If the state vector $x(1)$ is known and the gift funds $z(t)$, $t = 1,\ldots,N-1$ are predicted, then the control sequence $u(t)$, $t = 1,\ldots,N-1$ with (2) determines $x(t)$ for all future periods $t = 1,\ldots,N$. After making various assumptions about the

likely levels of state aid and gifts in the future, SCPM determines optimal enrollment, hiring, and construction policies for a given set of institutional objectives, which are described in the next section.

Student tuition could be included as a control variable instead of a predetermined factor. This would more accurately reflect the decisions of most private institutions and a growing number of public institutions. However, two major problems have to be dealt with to include tuition as a unconstrained control variable. A conceptual problem is the effect of additional tuition on student demand for attendance and on the quality of students able to pay the higher tuition.[12] A minor technical problem is the non-linearity of the money dynamics introduced by controllable tuition. The solution algorithm given in the Appendix will accommodate both linear and nonlinear dynamic systems.

However, this does raise the issue of the validity of the linearity assumption embodied in the SCPM systems dynamics. While in any specific implementation, the functional form of the dynamics would be an empirical question, there are several justifications for the use of linear dynamics in our example: (1) the ease of interpretation of coefficients in terms of transition probabilities, depreciation factors, faculty salaries, etc.; (2) the experimental ease of formulation and modification; and (3) the lack of any information of a more generally useful and accurate formulation.

---

[12] See Miller [1971] for a discussion of recent attempts to estimate student demand functions and Jewett [1971] for a presentation of a national student ability - willingness to pay model and analysis.

## CRITERION FUNCTION

The general form of the criterion function used by SCPM is

$$\min \left\{ J = P\big(x(N)\big) + \sum_{t=1}^{N-1} V_t\big(x(t),u(t),z(t)\big) \right\} \qquad (7)$$

where $P$ and $V_t$ are the relative quadratic loss functions derived in a previous section. In this study, $P$ is the sum of four quadratic terms and $V_t$ ($V_t$=V for all t) is a summation of nine quadratic terms. For purposes of exposition we separate $V$ into five sets of terms relating to the administrator's objectives expressed in terms of the student/faculty ratio, faculty mix, space requirements, student mix, and financial stability. $P$ will be discussed separately.

### Student/Faculty Ratio

One proxy measure of the amount and quality of student/faculty interaction at an institution is the ratio of students to its (FTE) faculty. SCPM enables a campus administrator to specify a targeted ratio and then seeks a set of controls which minimizes the deviation of the actual student/faculty ratio from the target. If we define

$r_1$ = target student/faculty ratio,

$k_1$ = some scalar weight,

$TF_t = \sum_{i=1}^{4} \left[x_i(t) + u_i(t)\right]$, which is the total faculty at time t, and

$TS_t = \sum_{i=5}^{8} \left[x_i(t) + u_i(t)\right]$, which is the total students at time t,

then the first term of $V_t$ may be written

$$k_1(r_1 TF_t - TS_t)^2 \quad . \qquad (i)$$

If an administrator wished, this term could be expanded to reflect students by level and faculty by rank to describe, for example, the exposure of lower division students to tenured faculty. Similar expansions are possible in most of the terms of the criterion function but were not reported here because they do not alter the solution procedure or the basic utility and results of the model.

## Faculty Mix

Another proxy criterion for assessing the quality or prestige of a college is the mix of its faculty by rank. [See Rowe, Wagner and Weathersby (1970).] In the case of community colleges, or any other cases for which there are no ranks but rather salary schedules, we may interpret the four (or fewer) levels of faculty purely in terms of salary. In any case, we assume that the administrator has preferences over different mixes of faculty by level and we allow him to specify target ratios which describe the desired mix.

If we define

$F_i(t) = x_i(t) + u_i(t)$, $i = 1,2,3,4$, which is the number of FTE faculty at level $i$ in year $t$,

$r_i$ = target ratios of each rank relative to the number of full professors, $i = 2,3,4$, and

$k_i$ = scalar weights $i = 2,3,4$,

then the next three terms of $V_t$ are

$$k_i\left(r_i F_1(t) - F_i(t)\right)^2 , \quad i = 2,3,4 . \tag{ii}$$

## Space

Typically space needs are largely determined by either student enrollment or faculty size or a combination of the two. SCPM makes the simplifying assumptions that space is interchangeable and available in continuously variable amounts. For a small, homogenous college, space interchangeability may not be a devastating assumption because in liberal arts subjects rooms of each type can be used by most disciplines even though one cannot easily interchange lecture halls and offices. The assumption of continuously variable space is a weakness of the model, because new construction occurs by project or building and, therefore, occurs in quantum jumps. However, we do include the time lag of construction from start to completion. Recalling that $x_{12}(t)$ is the available space at $t$, we let

$k_5$ be a scalar weight and

$c_1, c_2, c_3$ be space standard coefficients determining space needs as a linear combination of total faculty $(TF_t)$ and total students $(TS_t)$. The fifth term of $V_t$ is then

$$k_5\left(c_1 + c_2 TS_t + c_3\, TF_t - x_{12}(t)\right)^2 . \qquad \text{(iii)}$$

## Student Mix

Fiscal planning can be much more effective if student enrollments can be forecasted several years into the future. SCPM does not attempt to describe enrollments by discipline, although it could by defining additional state variables and equations; instead, SCPM focuses on student levels. Furthermore, for small colleges it was felt that average costs would not vary significantly across disciplines because these small colleges rarely can afford massive commitments of dollars to facilities and

faculties in the hard sciences and engineering, traditionally the most expensive disciplines. (Schools such as Cal Tech and MIT are clearly exceptions to this rule and they would need to recognize student discipline and level.) The next three terms of $V_t$ are constructed from the same pattern as for faculty.

$$k_i\left(r_i\ S_5(t) - S_i(t)\right)^2 \ , \ i = 6,7,8 \ , \text{ and } S_i = \text{students of level } i \ . \tag{iv}$$

## Monetary Balance

The last term of $V_t$ is a balance equation, expressed as a quadratic penalty function, which forces the annual net cash balance at the end of each period towards zero. Campus administrators are assumed to seek policies so that the cash inflow, e.g., transfers from endowment, gifts, and student revenues, is equal to the outflow, e.g., transfers to endowment, faculty salaries, construction costs, maintenance costs, and other operating costs. Otherwise, too much is withdrawn from income producing investments or, conversely, not enough is invested—both of which have an opportunity cost to the institution and should be avoided. The net cash balance is given by $x_{14}(t+1)$ ; however it is included in $V_t$ as a function of $x(t)$, $u(t)$, $z(t)$ . With

$k_9$ = a scalar weight and

$f_{14,j}$ , $g_{14,j}$ defined as before, the final expression in $V_t$ is

$$k_9\left(\sum_j f_{14,j}x_j(t) + \sum_j g_{14,j}u_j(t) + h_{14,13}z_{13}(t) + z_{14}(t)\right)^2 \ . \tag{v}$$

## Terminal Conditions

Because this optimal decision problem is formulated as a finite horizon differential dynamic programming problem, it is necessary to introduce "artificial" targets in the last planning period to correct for the truncated horizon. To prevent SCPM from "selling off" uncompleted space in the last three periods (space which is not usable during the model's lifetime, and therefore, space of no value to the institution) we drive $x_9(N), x_{10}(N), x_{11}(N)$ to zero by including in $P(N)$

$$k_{10} x_i(N)^2 \quad , \; i = 9,10,11 \; , \text{ where } k_{10} \text{ is a scalar weight.} \qquad \text{(vi)}$$

The last term of $P(N)$ is the net cash balance equation expressed as a function of $x(N)$, and write

$$k_9 x_{14}(N)^2 \quad , \; k_9 \text{ a scalar weight.} \qquad \text{(vii)}$$

When put in the form of equation (7), expressions (i) - (vii) constitute the criterion function currently being investigated.

As discussed previously, the scalar weights were chosen to equalize unit losses and thereby avoid inducing artificial minima. Table 3 shows the values for the scalar weights used in this study.

Although the target ratios were chosen to be constant over time, it would be possible to choose target paths showing the evolution of the variables over time and SCPM would then solve for the optimal decisions for this path. Another formulation of the criterion function would enable the campus planner to determine the shortest time in which the desired targets could be achieved. However, the numerical results reported in the next section employed equations (i) through (vii).

## SUMMARY OF RESULTS

To assess the realism of the model, a reference set of input data was developed which represents the operations of an "average" small liberal arts college. Actually, these data were modelled after data in the University of Santa Clara [1970]. Changes in this reference set (denoted Data Set 1) were made to reflect different operating policy decisions which might be made by campus administrators and to reflect different assumptions about future levels of external financial support. The optimal decisions for these new environments were then compared to the base case results. (See Tables 2 and 3 for a summary of Data Set 1).

Additionally, the college's optimal decision problem was solved for only non-negative values for the control variables. The resulting values thereby achieve sub-optimal solutions which trade optimally for more realistic results. It was shown that these solutions achieve the targeted ratios and provide adequate space as specified by the objective function but that they result in a much greater variation in the net cash balances at the end of each operating period. (See Table 5 for a comparison of net cash balances.)

Student admissions, faculty hiring, and new construction decisions are the normal outputs of the model. In addition, the model can answer questions about the operational effects of alternative funding methods which are of considerable interest to external funding agencies such as state and local governments. To examine these questions, three additional data sets were analyzed from this policy perspective.

TABLE 2

Accounting Summary for SCPM Example*
(Enrollment Level: 800)

| EXPENSE | | % of Total | INCOME | | % of Total |
|---|---|---|---|---|---|
| Instruction | $ 104,400 | 6.39 | Tuition & Fees | $ 970,400 | 58.07 |
| Student Aid | 119,200 | 7.31 | Transfers from Endowment | 115,200 | 6.89 |
| Student Services | 266,800 | 16.33 | | | |
| Total Student Related | $ 490,400 | 30.03 | Gifts & Grants | 364,000 | 21.78 |
| Salaries: Teaching Faculty | 418,000 | 25.60 | Other (gov't. aid) | 221,600 | 13.26 |
| Salaries: Admin./Other Fac. | 222,000 | 13.60 | | | |
| Total Fac./Admin. Salaries | $ 640,000 | 39.20 | TOTAL | $1,671,200 | 100.00 |
| Plant M&O | 208,000 | 12.73 | | | |
| Plant Additions | 294,450 | 18.04 | | | |
| Total Plant | $ 502,450 | 30.77 | | | |
| TOTAL | $1,632,850 | 100.00 | | | |

*This accounting summary was derived from "An Introduction to Program Planning Budgeting and Evaluation for Colleges and Universities" - July 1970 - University of Santa Clara--Office of Institutional Planning.

Additional Assumptions:

1) Eighty percent of instructional costs are faculty salaries;
2) There are $3,840,000 of restricted funds yielding 3% per annum;
3) Plant M&O costs $2/ASF; there are 130 ASF/student;
4) Student/faculty ratio is 16/1; faculty includes teaching, research and administrative staff exclusive of support.

TABLE 3

Data Set 1

F-MATRIX

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.93 | 0.05 | | | | | | | | | | | | |
| 2 | | 0.85 | 0.05 | | | | | | | | | | | |
| 3 | | | 0.80 | 0.50 | | | | | | | | | | |
| 4 | | | | 0.10 | | | | | | | | | | |
| 5 | | | | | | | | | | | | | | |
| 6 | | | | | 0.95 | 0.02 | | | | | | | | |
| 7 | | | | | | 0.70 | 0.03 | | | | | | | |
| 8 | | | | | | | 0.70 | 0.04 | | | | | | |
| 9 | | | | | | | | | | | | | | |
| 10 | | | | | | | | | 1.00 | | | | | |
| 11 | | | | | | | | | | 1.00 | | | | |
| 12 | | | | | | | | | | | 1.00 | 0.98 | | |
| 13 | | | | | | | | | | | | | 1.00 | |
| 14 | -1.7 | -1.4 | -1.1 | -0.8 | 0.06 | 0.06 | 0.06 | 0.06 | -4.5 | -4.5 | -2.0 | -0.2 | 0.03 | 1.00 |

G-MATRIX

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.70 | | | | | | | | |
| 2 | | 0.70 | | | | | | | |
| 3 | | | 0.95 | 0.30 | | | | | |
| 4 | | | | 0.10 | | | | | |
| 5 | | | | | 0.03 | | | | |
| 6 | | | | | 0.60 | 0.02 | | | |
| 7 | | | | | | 0.70 | 0.03 | | |
| 8 | | | | | | | 0.70 | 0.04 | |
| 9 | | | | | | | | | 1.00 |
| 10 | | | | | | | | | |
| 11 | | | | | | | | | |
| 12 | | | | | | | | | |
| 13 | | | | | | | | | |
| 14 | -1.7 | -1.4 | -1.1 | -0.8 | 0.06 | 0.06 | 0.06 | 0.06 | -1.5 |

TABLE 3 (continued)

Vector of Initial x-Values

| x(1) | x(2) | x(3) | x(4) | x(5) | x(6) | x(7) | x(8) | x(9) | x(10) | x(11) | x(12) | x(13) | x(14) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10. | 12. | 15. | 1.0 | 10. | 170. | 140. | 130. | 2.08 | 2.08 | 2.08 | 104. | 384. | 0.0 |

Vector of Predetermined Funds ($10,000's)

| Per. | Amount | Per. | Amount | Per. | Amount | Per. | Amount |
|---|---|---|---|---|---|---|---|
| 1) | 58.56 | 6) | 61.55 | 11) | 64.69 | 16) | 67.99 |
| 2) | 59.15 | 7) | 62.17 | 12) | 65.34 | 17) | 68.67 |
| 3) | 59.74 | 8) | 62.79 | 13) | 65.99 | 18) | 69.36 |
| 4) | 60.34 | 9) | 63.42 | 14) | 66.65 | 19) | 70.05 |
| 5) | 60.94 | 10) | 64.05 | 15) | 67.32 | | |

TARGETS AND WEIGHTS

| Item | Target | Weight | Item | Target | Weight |
|---|---|---|---|---|---|
| Assoc/Full: | 1.5 | 1.0 | Soph/Frosh: | 0.7 | 1.0 |
| Asst/Full: | 2.0 | 1.0 | Fr./Frosh: | 0.65 | 1.0 |
| Inst/Full: | 0.5 | 1.0 | Sr./Frosh: | 0.5 | 1.0 |
| Stu/Faculty: | 16.0 | 0.01 | ASF/Student: | 130. | 50.0 |
| Net Cash Bal: | 0.0 | 0.5 | | | |

## Comparison of Optimal versus Sub-Optimal Results

The general form of the optimal control problem is to

$$\underset{u(t)}{\text{Min}} \left\{ J = P\big(x(N)\big) + \sum_{t=0}^{N-1} V\big(x(t),u(t),z(t),t\big) \right\}$$

subject to

$$x(0) : \text{known}$$
$$x(t+1) = f\big(x(t),u(t),z(t),t\big) \ .$$

This formulation does not constrain the sign of either the state or control variables. This is the form for which the solution algorithm given in the Appendix was designed. While firing full professors and selling newly constructed space may appeal to some interests, negative values for control variables are in general not meaningful. Furthermore, our study revealed that unconstrained solutions to the optimal policy problem usually contained several such negative decision values, which, while small in magnitude, were nevertheless inappropriate and unrealistic.

Rather than impose inequality constraints which would require a reformulation of the model or attach penalty functions to the criterion function to facilitate the use of a sequential optimization algorithm, we initially included a switch in the computer program which set negative values of the computed control variables to zero within the iteration sequence.[13] Since the algorithm computes the improved $u(t)$ values based on small variations in $x(t)$ and since those variations are not substantially altered by zeroing out negative values of $u(t-1)$, the

[13] One can solve the general formulation with inequality constraints at considerable additional complexity and expense, see Jacobson [1969], but we concluded that at this stage of development these refinements were not worth the additional cost because the constrained and unconstrained results were so similar.

resulting sub-optimal path should provide acceptable solutions. A comparison of the two sets of results confirms this assertion.

Three sets of comparison runs were made to determine the loss from sub-optimization. These runs were based on the original Data Set 1, Data Set 2 in which exogenous funds grow at 2% per year, and Data Set 3 in which per-student income increased by $100 per year. Tables 4 and 5 show the differences in control variables and in the yearly net cash balances for Data Set 1. Similar results obtained for Data Sets 2 and 3. The differences between the two sets of control variable solutions for Data Set 1 are summarized below.

| Control Variable | Mean Value Unconstrained | Mean Value Constrained | Mean of The Absolute Differences |
|---|---|---|---|
| Full Professors | 0.12 | 0.15 | 0.25 |
| Assoc. Professors | 1.86 | 1.85 | 0.63 |
| Asst. Professors | 2.26 | 2.35 | 0.73 |
| Instructors | 4.74 | 4.88 | 0.30 |
| Freshmen | 286.39 | 287.52 | 5.03 |
| Sophomores | 23.37 | 23.75 | 5.39 |
| Juniors | 42.22 | 42.62 | 4.73 |
| Seniors | 8.12 | 8.36 | 4.01 |
| New Construction (000's ASF) | 2.92 | 2.94 | 3.41 |

It is clear from an examination of Table 6 that in terms of the targeted values specified by the objective function, there is very little loss of utility associated with using the constrained formulation. Because the negative values for the control variables generated by the unconstrained solutions are unrealistic, we shall concentrate on the constrained, sub-optimal results for the balance of this discussion.

TABLE 4

Comparison of Control Variables
Data Set 1
Unconstrained vs Non-Negative Modes

| | NEW FACULTY | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Unconstrained | | | | Non-Negative | | | |
| Per. | Prof. | Assoc. | Asst. | Inst. | Prof. | Assoc. | Asst. | Inst. |
| 1 | 0.10 | 3.20 | 5.11 | 3.82 | 0.10 | 3.20 | 5.11 | 3.82 |
| 2 | -0.03 | 1.68 | 1.43 | 4.63 | 0.00 | 1.68 | 1.43 | 4.63 |
| 3 | 0.12 | 1.75 | 2.25 | 4.43 | 0.00 | 1.48 | 2.10 | 4.66 |
| 4 | 0.02 | 1.54 | 2.03 | 4.53 | 0.16 | 1.89 | 2.19 | 4.37 |
| 5 | 0.39 | 2.12 | 2.78 | 4.67 | 0.31 | 1.94 | 2.67 | 4.77 |
| 6 | -0.21 | 1.23 | 1.36 | 4.47 | 0.00 | 1.14 | 1.50 | 4.85 |
| 7 | 0.43 | 2.20 | 3.13 | 4.73 | 0.12 | 2.08 | 2.45 | 4.38 |
| 8 | -0.06 | 1.46 | 1.62 | 4.63 | 0.32 | 2.00 | 2.78 | 4.93 |
| 9 | 0.22 | 1.92 | 2.60 | 4.66 | 0.00 | 0.38 | 1.08 | 5.68 |
| 10 | 0.10 | 1.71 | 2.21 | 4.80 | 0.00 | 1.38 | 1.77 | 5.24 |
| 11 | 0.18 | 1.90 | 2.36 | 4.69 | 0.97 | 4.01 | 4.18 | 4.49 |
| 12 | 0.05 | 1.66 | 2.18 | 4.79 | 0.00 | 1.14 | 1.33 | 5.14 |
| 13 | 0.25 | 1.98 | 2.62 | 4.90 | 0.00 | 0.62 | 1.53 | 5.50 |
| 14 | 0.12 | 1.84 | 2.14 | 4.72 | 0.00 | 1.73 | 2.17 | 5.18 |
| 15 | 0.00 | 1.56 | 2.28 | 5.02 | 0.53 | 3.25 | 3.64 | 4.95 |
| 16 | 0.53 | 2.53 | 3.07 | 4.89 | 0.00 | 1.05 | 1.36 | 5.47 |
| 17 | -0.42 | 0.96 | 1.13 | 4.83 | 0.00 | 1.70 | 2.05 | 4.88 |
| 18 | 0.77 | 2.81 | 4.12 | 5.33 | 0.39 | 2.67 | 3.40 | 5.04 |
| 19 | -0.27 | 1.37 | 0.87 | 4.56 | 0.02 | 1.83 | 1.89 | 4.77 |

| | NEW STUDENTS | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Unconstrained | | | | Non-Negative | | | |
| Per. | Frosh | Soph. | Jr. | Sr. | Frosh | Soph. | Jr. | Sr. |
| 1 | 270.8 | 26.5 | 42.5 | 10.4 | 270.7 | 26.5 | 42.5 | 10.4 |
| 2 | 272.8 | 20.7 | 39.5 | 7.1 | 272.7 | 20.8 | 39.6 | 7.1 |
| 3 | 272.8 | 21.4 | 39.5 | 7.1 | 272.8 | 21.4 | 39.5 | 7.1 |
| 4 | 272.8 | 21.3 | 39.5 | 7.0 | 272.8 | 21.3 | 39.5 | 7.0 |
| 5 | 282.1 | 27.9 | 45.6 | 11.7 | 282.4 | 28.0 | 45.7 | 11.8 |
| 6 | 271.9 | 15.2 | 34.4 | 2.3 | 276.3 | 18.1 | 37.1 | 4.4 |
| 7 | 285.5 | 30.5 | 48.0 | 13.7 | 277.6 | 22.3 | 40.7 | 7.7 |
| 8 | 279.4 | 18.4 | 37.6 | 4.5 | 289.1 | 29.8 | 47.7 | 12.9 |
| 9 | 284.3 | 25.1 | 43.5 | 9.5 | 282.8 | 18.5 | 37.9 | 4.5 |
| 10 | 286.3 | 23.8 | 42.5 | 8.4 | 278.9 | 19.2 | 38.3 | 5.2 |
| 11 | 287.9 | 23.5 | 42.5 | 8.2 | 301.4 | 37.4 | 54.9 | 18.4 |
| 12 | 288.1 | 22.7 | 41.8 | 7.5 | 294.5 | 19.4 | 39.5 | 4.6 |
| 13 | 294.1 | 26.7 | 45.6 | 10.4 | 288.6 | 18.7 | 38.7 | 4.6 |
| 14 | 292.2 | 21.8 | 41.4 | 6.7 | 289.1 | 22.7 | 42.0 | 7.6 |
| 15 | 294.5 | 24.4 | 43.8 | 8.7 | 307.0 | 35.2 | 53.5 | 16.4 |
| 16 | 304.2 | 29.8 | 48.9 | 12.5 | 300.2 | 19.7 | 40.3 | 4.7 |
| 17 | 289.5 | 13.8 | 34.6 | 0.6 | 294.2 | 19.1 | 39.5 | 4.6 |
| 18 | 315.3 | 40.2 | 58.4 | 20.1 | 309.3 | 33.4 | 52.3 | 15.1 |
| 19 | 294.0 | 10.4 | 32.2 | -2.2 | 302.5 | 19.8 | 40.6 | 4.8 |

TABLE 5

Comparison of Unrestricted Funds Balances

Unconstrained vs Non-Negative Modes
(Data Set 1)

| Per. | Unconstrained | Non-Negative |
|---|---|---|
| 1 | $ 15,240. | $ 15,220. |
| 2 | -24,096. | - 47,390. |
| 3 | 13,195. | 16,260. |
| 4 | - 5,572. | 26,660. |
| 5 | 856. | 54,500. |
| 6 | 787. | 14,030. |
| 7 | 678. | 45,840. |
| 8 | - 4,560. | -139,320. |
| 9 | 9,166. | - 97,000. |
| 10 | -11,996. | 146,450. |
| 11 | 10,543. | 109,970. |
| 12 | - 3,486. | -128,660. |
| 13 | - 8,290. | - 52,060. |
| 14 | 20,927. | 101,710. |
| 15 | -28,550. | 23,120. |
| 16 | 25,150. | - 11,670. |
| 17 | - 7,584. | 16,080. |
| 18 | -21,526. | 7,077. |
| 19 | 25,923. | 14,157. |
| NET | $ 6,833. | $ 5,974. |

Comparison of Control Variables

Unconstrained vs Non-negative Modes
(Data Set 1)

New Construction

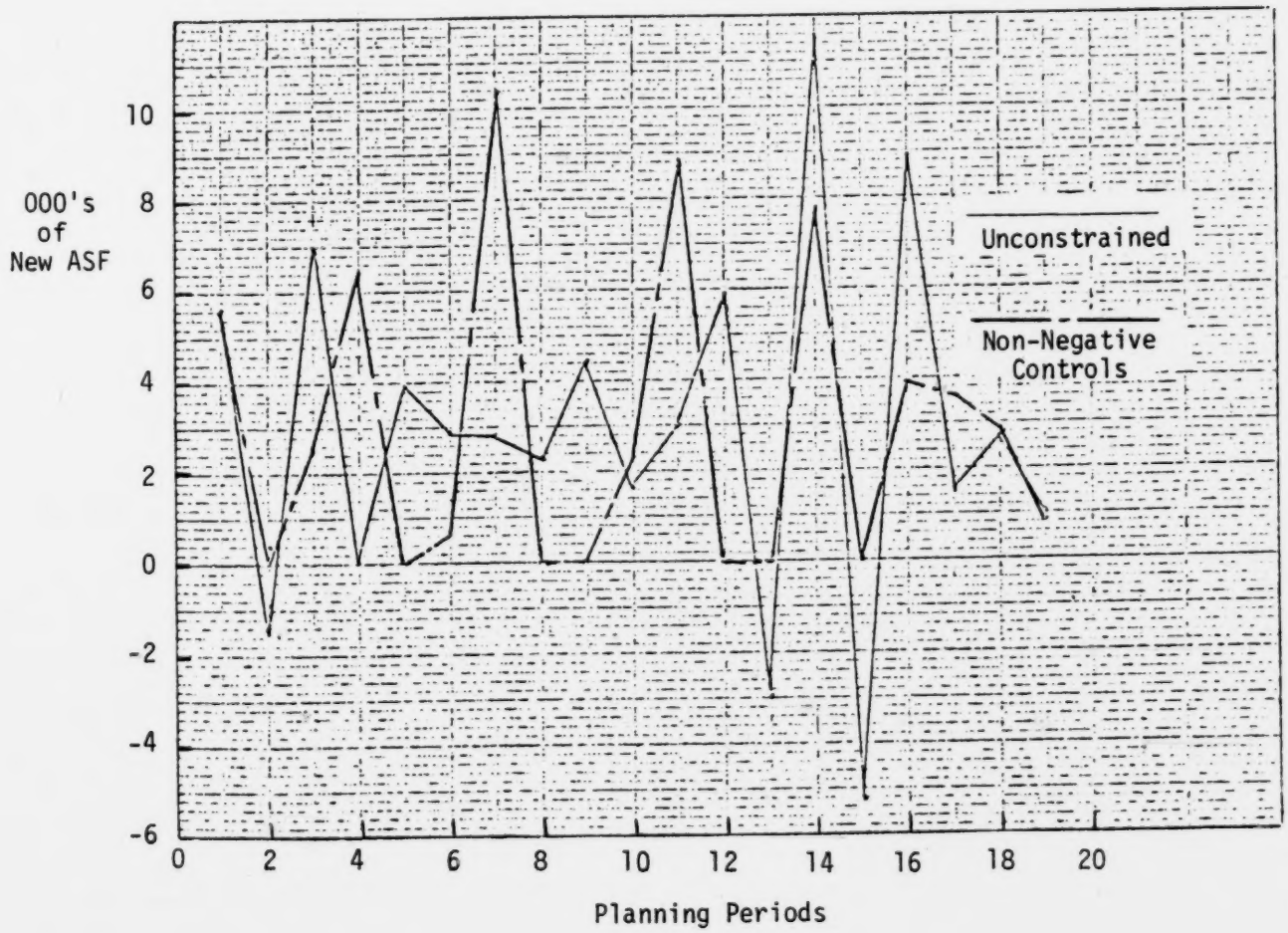


FIGURE 4

TABLE 6

Comparison of Unconstrained (A) vs Non-Negative (B) Modes
Preference Function Targets
(Data Set 1)

| Period | Associates / Professors | | Assistants / Professors | | Instructors / Professors | | Sophomores / Freshmen | | Juniors / Freshmen | | Seniors / Freshmen | | Students / Faculty | | ASF Required / ASF Built | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B |
| Target | 1.500 | 1.500 | 2.000 | 2.000 | 0.500 | 0.500 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 16.00 | 16.00 | 1.000 | 1.000 |
| 1 | 1.506 | 1.506 | 1.992 | 1.992 | 0.478 | 0.478 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 15.93 | 15.93 | 1.000 | 1.000 |
| 2 | 1.496 | 1.492 | 2.005 | 1.999 | 0.514 | 0.513 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 16.06 | 16.05 | 1.001 | 1.001 |
| 3 | 1.502 | 1.490 | 1.997 | 2.003 | 0.493 | 0.521 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 15.99 | 16.08 | 1.001 | 1.001 |
| 4 | 1.499 | 1.504 | 2.001 | 1.995 | 0.503 | 0.485 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 16.03 | 15.97 | 1.001 | 1.001 |
| 5 | 1.500 | 1.497 | 2.000 | 2.004 | 0.500 | 0.511 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 16.00 | 16.05 | 1.000 | 1.001 |
| 6 | 1.500 | 1.460 | 2.000 | 1.973 | 0.499 | 0.528 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 16.00 | 16.08 | 1.000 | 1.001 |
| 7 | 1.500 | 1.505 | 2.000 | 1.992 | 0.500 | 0.478 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 16.01 | 15.95 | 1.001 | 1.001 |
| 8 | 1.499 | 1.496 | 2.001 | 2.006 | 0.503 | 0.516 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 16.03 | 16.07 | 1.001 | 1.001 |
| 9 | 1.501 | 1.383 | 1.998 | 1.934 | 0.495 | 0.599 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 15.98 | 16.26 | 1.000 | 1.001 |
| 10 | 1.498 | 1.398 | 2.003 | 1.929 | 0.508 | 0.565 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 16.04 | 16.16 | 1.001 | 1.001 |
| 11 | 1.502 | 1.512 | 1.998 | 1.982 | 0.494 | 0.451 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 15.98 | 15.85 | 1.000 | 1.001 |
| 12 | 1.499 | 1.460 | 2.001 | 1.967 | 0.503 | 0.518 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 16.01 | 16.05 | 1.000 | 1.001 |
| 13 | 1.499 | 1.378 | 2.002 | 1.902 | 0.505 | 0.558 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 16.02 | 16.11 | 1.000 | 1.001 |
| 14 | 1.503 | 1.415 | 1.996 | 1.928 | 0.488 | 0.534 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 15.97 | 16.06 | 1.000 | 1.001 |
| 15 | 1.495 | 1.503 | 2.007 | 1.996 | 0.518 | 0.490 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 16.08 | 15.98 | 1.001 | 1.000 |
| 16 | 1.504 | 1.440 | 1.994 | 1.962 | 0.485 | 0.544 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 15.96 | 16.12 | 1.001 | 1.001 |
| 17 | 1.498 | 1.459 | 2.003 | 1.952 | 0.507 | 0.497 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 16.03 | 15.97 | 1.000 | 1.001 |
| 18 | 1.497 | 1.502 | 2.004 | 1.997 | 0.511 | 0.492 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 16.05 | 15.99 | 1.001 | 1.001 |
| 19 | 1.507 | 1.506 | 1.989 | 1.990 | 0.470 | 0.474 | 0.700 | 0.700 | 0.650 | 0.650 | 0.500 | 0.500 | 15.92 | 15.93 | 1.001 | 1.001 |

## Comparison of Results from Various Policy Alternatives

Perhaps the most striking feature of the solutions generated by the model is that they do not show a smooth expansion path, either in terms of total enrollments or in terms of any of the control variables. This is not surprising mathematically, but it may surprise administrators unfamiliar with controllable, dynamic systems with different response times.

Intuitively, what has happened to the reference data set for the constrained case is the following. The preference function desires the ratio of available ASF per student to be fixed at 130. Enrollments are therefore constrained during the initial four years by the amount of physical space under construction at the beginning of the planning period, ($x_9$,$x_{10}$, and $x_{11}$ at t = 1). For the reference data set, these are set at a level which exactly counteracts the depreciation of the existing capital stock. For the first four years therefore, the physical space available remains constant. Because faculty is linked to enrollments through the Student/Faculty Ratio, it is unnecessary to spend operating dollar balances resulting from the 1% growth in outside funds, to increase the size of the faculty during this period. As a result, the model spends any "excess funds" on new capital construction. This becomes available after the fourth year at which time enrollments begin to increase.

Although students yield a net dollar gain from the tuition level, they induce costs in the form of faculty salaries, capital needs, etc., so that it is impossible to pay for additional space by simply adding students to the rolls. When enrollments increase, therefore, the amount of new construction must decline relatively. This will eventually cause some decline in enrollment levels from peak periods, and thus induce a cyclical pattern of physical expansion and student growth.

Although the system dynamics and the solution process are considerably more complex, the foregoing is the predominant reason for the cyclical variability in the expansion path. Another reason is the different time constants or response times for the various state variables. Full professors spend a longer average time in the system than instructors, freshmen more than entering seniors, and so forth. The time behavior of aggregates is built up from many of these overlapping transients and, therefore, the aggregates show a cyclical time behavior.

Data Sets 1 and 2 differ only in the assumptions concerning the levels of external aid, with a 1% yearly increase in gifts, grants and government assistance reflected in Data Set 1 and a 2% yearly increase contained in Data Set 2. Both employ the same levels of aid for the initial year. At enrollment levels of 800 students, these funds account for 35% of current income in the initial year and 36.02% and 37.91% respectively for the total planning horizon. A comparison of the enrollment patterns generated is shown in Table 7. As can be seen from the table, there is no substantial increase in enrollments until the 10th period despite the fact that an additional $225,700 has been received by the institution through the 9th period. Since the 19-year "marginal-cost" of increased enrollments is approximately $5,500, one might intuitively expect that a smooth expansion path would be generated which should have enrolled an additional 41 students through the first 10 years. In fact, the school has only enrolled an additional 3 students.

Another way of looking at the data is to examine the increase in external funds in relation to the total income over the 19-year period. Here it can be seen that the additional $1.1876 million generated by Data

TABLE 7

Comparison of Data Set 1 and Data Set 2
Enrollments and Costs

| Period | Data Set 1 Enrollment | Data Set 2 Enrollment | DS2 -- DS1 Difference | Data Set 1 Exog.Funds* | Data Set 2 Exog.Funds | DS2 -- DS1 Difference | % Change Enroll. | % Change* Exog.Funds |
|---|---|---|---|---|---|---|---|---|
| 1 | 800 | 800 | 0 | 585,600 | 585,600 | 0 | 0.00 | 0.00 |
| 2 | 801 | 800 | -1 | 591,500 | 597,300 | 5,800 | -0.12 | 0.98 |
| 3 | 801 | 800 | -1 | 597,400 | 609,300 | 11,900 | -0.12 | 1.99 |
| 4 | 801 | 800 | -1 | 603,400 | 621,400 | 18,000 | -0.12 | 2.98 |
| 5 | 828 | 828 | 0 | 609,400 | 633,900 | 24,500 | 0.00 | 4.02 |
| 6 | 812 | 812 | 0 | 615,500 | 646,500 | 31,000 | 0.00 | 5.03 |
| 7 | 815 | 820 | 5 | 621,700 | 659,500 | 37,800 | 0.61 | 6.08 |
| 8 | 848 | 848 | 0 | 627,900 | 672,700 | 44,800 | 0.00 | 7.13 |
| 9 | 831 | 832 | 1 | 634,200 | 686,100 | 51,900 | 0.12 | 8.18 |
| 10 | 819 | 842 | 23 | 640,500 | 699,800 | 59,300 | 2.80 | 9.25 |
| 11 | 883 | 876 | -7 | 646,900 | 713,800 | 66,900 | -0.79 | 10.34 |
| 12 | 865 | 858 | -7 | 653,400 | 728,100 | 74,700 | -0.80 | 11.43 |
| 13 | 848 | 860 | 12 | 659,900 | 742,700 | 82,800 | 1.41 | 12.54 |
| 14 | 848 | 916 | 68 | 666,500 | 757,500 | 91,000 | 8.01 | 13.65 |
| 15 | 900 | 897 | -3 | 673,200 | 772,700 | 99,500 | -0.33 | 14.78 |
| 16 | 882 | 880 | -2 | 679,900 | 788,100 | 108,200 | -0.22 | 15.96 |
| 17 | 864 | 938 | 74 | 686,700 | 803,900 | 117,200 | 8.56 | 17.06 |
| 18 | 906 | 921 | 15 | 693,600 | 820,000 | 126,400 | 1.65 | 18.22 |
| 19 | 889 | 930 | 41 | 700,500 | 836,400 | 135,900 | 4.61 | 19.40 |
| Total | 16,041 | 16,258 | 217 | 12,187,700 | 13,375,300 | 1,187,600 | 1.35 | 9.74 |

*Exogenous funds constitute approximately 36% of current income, so that a 9.74% increase in exogenous funds represents a 3.5% increase in current income.

Set 2 is 3.51% of the total income generated via Data Set 1. This 3.5% increase in total income thus would result in a 1.35% increase in total enrollments if it were to come in the form of additional gifts, grants, and government aid according to this exponentially increasing function of time.

Suppose instead the increase in income had come from tuition charges. Data Set 3 simulates the effect of a $100 increase in tuition charges (or equivalently, an additional subsidy of $100 per student) by changing the net return from students from $600 to $700. (Recall that the coefficients $f_{14,j}$ and $g_{14,j}$ , j=5-8 , are the difference between tuition and average student-related costs such as admissions, counseling, student-aid and health facilities.)

If the institution were to increase tuition and fees by $100, neglecting per-student student aid increases, enrollments would increase 3.57% over the 19-year period. The increase in total income represented by this policy change is $2,356,449 or approximately 7%. Table 8 shows a comparison of enrollments and income under the assumptions of Data Sets 1 and 3.

Looking at these results from the point of view of a potential fundor, such as the State or Federal government, it would appear at first blush that the most productive means of funding the institution would be the per-student institutional subsidy. In order to investigate the question more thoroughly, Data Set 4 was developed. This data set increases the external inputs by an amount equal to the $100/student-additional income generated by the increase in tuition at optimal enrollment levels. Table 9 shows the enrollments generated by Data Sets 2, 3 and 4 and the marginal costs of enrollments. A comparison of these costs shows that for the purpose of increasing enrollments, it is more effective to fund the institution directly

TABLE 8

Comparison of Data Set 1 and Data Set 3
Enrollments and Costs

| Period | Data Set 1 Enrollment | Data Set 3 Enrollment | DS3 -- DS1 Difference | Additional Income Generated by Tuition Increase | % Change Enrollments | % Change* Tuition Income |
|---|---|---|---|---|---|---|
| 1 | 800 | 800 | 0 | 80,000 | 0.00 | 0.00 |
| 2 | 801 | 800 | -1 | 78,787 | -0.12 | 8.10 |
| 3 | 801 | 800 | -1 | 78,787 | -0.12 | 8.10 |
| 4 | 801 | 801 | 0 | 80,100 | 0.00 | 8.24 |
| 5 | 828 | 858 | 30 | 122,190 | 3.62 | 12.16 |
| 6 | 812 | 841 | 29 | 119,277 | 3.57 | 12.10 |
| 7 | 815 | 824 | 9 | 93,317 | 1.10 | 9.43 |
| 8 | 848 | 870 | 22 | 113,686 | 2.59 | 11.05 |
| 9 | 831 | 860 | 29 | 121,177 | 3.48 | 12.02 |
| 10 | 819 | 861 | 42 | 137,046 | 5.12 | 13.79 |
| 11 | 883 | 904 | 21 | 115,873 | 2.37 | 10.81 |
| 12 | 865 | 886 | 21 | 114,073 | 2.42 | 10.87 |
| 13 | 848 | 885 | 37 | 133,381 | 4.36 | 12.96 |
| 14 | 848 | 943 | 95 | 209,535 | 11.20 | 20.37 |
| 15 | 900 | 924 | 24 | 121,512 | 2.66 | 11.13 |
| 16 | 882 | 905 | 23 | 118,399 | 2.60 | 11.06 |
| 17 | 864 | 961 | 97 | 213,761 | 11.22 | 20.39 |
| 18 | 906 | 943 | 37 | 139,181 | 4.08 | 12.66 |
| 19 | 889 | 948 | 59 | 166,367 | 6.63 | 15.42 |
| Total | 16,041 | 16,614 | 573 | 2,356,449 | 3.57 | 12.11 |

*Tuition and fees account for approximately 58% of current income, i.e., a 12% change in tuition income is approximately a 7% change in total current income.

TABLE 9

Comparison of Data Set 1, Data Set 2, Data Set 3, and Data Set 4
Enrollments and Marginal Costs

| | Data Set 1 | Data Set 2 | | | Data Set 3 | | | Data Set 4 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Period | Enrollment | Enrollment | Change in Exog.Funds | Marginal Cost | Enrollment | Subsidy= $100/Stu. | Marginal Cost | Enrollment | Change in Exog.Funds | Marginal Cost |
| 1 | 800 | 800 | 0 | 0 | 800 | 80,000 | ∞ | 800 | 80,000 | ∞ |
| 2 | 801 | 800 | 5,800 | - 5,800 | 800 | 80,000 | -160,000 | 801 | 80,000 | ∞ |
| 3 | 801 | 800 | 11,900 | - 8,850 | 800 | 80,000 | -120,000 | 801 | 80,000 | ∞ |
| 4 | 801 | 800 | 18,000 | -11,900 | 801 | 80,100 | -160,000 | 801 | 80,100 | ∞ |
| 5 | 828 | 828 | 24,500 | -20,067 | 858 | 85,800 | 14,496 | 858 | 85,800 | 13,530 |
| 6 | 812 | 812 | 31,000 | -30,400 | 841 | 84,100 | 8,596 | 841 | 84,100 | 8,305 |
| 7 | 815 | 820 | 37,800 | 64,500 | 824 | 82,400 | 8,673 | 824 | 82,400 | 8,418 |
| 8 | 848 | 848 | 44,800 | 86,900 | 870 | 87,000 | 7,493 | 870 | 87,000 | 7,327 |
| 9 | 831 | 832 | 51,900 | 75,233 | 860 | 86,000 | 6,371 | 863 | 86,000 | 6,110 |
| 10 | 819 | 842 | 59,300 | 10,961 | 861 | 86,100 | 5,230 | 859 | 86,100 | 5,133 |
| 11 | 883 | 876 | 66,900 | 18,521 | 904 | 90,400 | 5,122 | 908 | 90,400 | 4,930 |
| 12 | 865 | 858 | 74,700 | 35,550 | 886 | 88,600 | 5,027 | 890 | 88,600 | 4,767 |
| 13 | 848 | 860 | 82,800 | 21,225 | 885 | 88,500 | 4,618 | 875 | 88,500 | 4,598 |
| 14 | 848 | 916 | 91,000 | 6,526 | 943 | 94,300 | 3,583 | 959 | 94,300 | 3,409 |
| 15 | 900 | 897 | 99,500 | 7,864 | 924 | 92,400 | 3,601 | 940 | 92,400 | 3,297 |
| 16 | 882 | 880 | 108,200 | 9,288 | 905 | 90,500 | 3,622 | 921 | 90,500 | 3,208 |
| 17 | 864 | 938 | 117,200 | 5,747 | 961 | 96,100 | 3,087 | 915 | 96,100 | 3,067 |
| 18 | 906 | 921 | 126,400 | 5,976 | 943 | 94,300 | 3,048 | 999 | 94,300 | 2,734 |
| 19 | 889 | 930 | 135,900 | 5,473 | 948 | 94,800 | 2,899 | 979 | 94,800 | 2,506 |
| Total | 16,041 | 16,258 | 1,187,600 | 5,473 | 16,614 | 1,661,400 | 2,899 | 16,704 | 1,661,400 | 2,506 |

rather than paying a per-student amount. The same subsidy, in terms of both total dollars and timing, has been assumed by Data Sets 3 and 4, yet the solution to the planning problem generates greater enrollments when the subsidy is given as a flat grant.[14]

## Conclusion

These examples show that realistic and relevant results can be obtained relatively easily and inexpensively by SCPM. The control theoretic approach both incorporates the multi-level, multi-decision maker hierarchical structures of higher education and enables educational planners to derive improved institutional plans and to evaluate many alternative operating policies. The robustness and flexibility of SCPM suggest that it could make a major contribution to improved educational planning.

---

[14]This counterintuitive result does not hold for the unconstrained case, if the optimal value of the criterion function is zero. If it is possible to achieve all the targets exactly in all planning periods, then MIN J = 0 and the optimal enrollments for Data Sets 3 and 4 would be identical. An alternative view of student tuition models and the effects of government subsidies is given in Weathersby [1970]. The above conclusion is supported in this supply and demand analysis.

# APPENDIX

## Solution Procedure

The linear-quadratic minimization problem is solved in one step using an adaptation of an algorithm devised by David Mayne [1966]. The following computer program is based on this algorithm.

A non-optimal trajectory is generated using a nominal control sequence. The effect on the criterion (penalty) function of small variations of the control sequence is determined. This enables an improved sequence to be chosen. In the case of a quadratic criterion and linear system dynamics, the first improved sequence is optimal.

The advantage of Mayne's approach over conventional dynamic programming approaches lies in immense reduction of core requirements. In place of the optimal return function $V^\circ$ of Bellman, Mayne uses $\nabla V^\circ$, the optimal variation in the non-optimal return function due to variation of the state variable. $\nabla V^\circ$ is expanded in a power series (to second order) of the variation in $x$, and difference equations are derived for the coefficients of the series. It is the identification of these coefficients which provide, in analytic form, the optimal change in $x(t)$, and hence, by working backward in time, of $u(t - 1)$.

The program, in its present form handling 25 variables and 20 planning periods, requires approximately 50K bytes of core and 1.5 minutes CPU time on IBM's 360-65 O.S. On the University of California Administrative Data Processing System this costs approximately $10.00 for an adequate numerical solution for one case. It is hoped that future versions will reduce the size and cost of running this program.

# COMPUTER PROGRAM

```
FORTRAN IV G LEVEL  18                MAIN                DATE = 71300          10/27/59

                C
                C     THIS PROGRAM EMPLOYS AN ITERATIVE PROCEDURE TO FIND A SEQUENCE
                C  OF CONTROL VECTORS WHICH MINIMIZES A NON-LINEAR N-PERIOD OBJECTIVE
                C  FUNCTION SUBJECT TO LINEAR-DYNAMIC CONSTRAINTS.  THE PROGRAM IS
                C  BASED ON AN ALGORITHM DEVELOPED BY DAVID MAYNE, PUBLISHED BY
                C  INT. JOURNAL OF CONTROL (1966).
                C
0001                 COMMON X(14,20),U(09,19),NX,NU,NPP,VX(14),   VXX(14,14),VUX(9,14),
                    1VUU(9,9),VXX2(14,14),VU(9),LPP,VX2(14),V(20),F(14,14),G(14,9),
                    2IIN,IOUT,NUMRUN
0002                 REAL JJ,OPT
0003                 DIMENSION A(14,14),B(9,14),ALPHA(9,19),BETA(9,14,19),HU(9),
                    1H(14,14),Z(14,19),XNEW(14,20),DELU(9),W(14,14),
                    2C(9,9),CINV(9,9),CC(45)
0004                 DOUBLE PRECISION C,CINV     ,CC
0005                 IIN=1
0006                 IOUT=6
                C
                C  READ BASIC DATA
                C
0007                1 READ(IIN,900) NUMIT,LPRNT,NU,NX,NPP,NUMRUN,OPT
0008                  IF (NUMIT.GT.99) STOP
0009                  LPP=NPP+1
0010                  IF(NUMRUN.GT.1) GO TO 150
0011                2 DO 100 I=1,NX
0012              100 READ(IIN,901)(F(I,J),J=1,NX)
0013                  IF(NUMRUN.GT.1) GO TO 150
0014                3 DO 101 I=1,NX
0015              101 READ(IIN,901)(G(I,J),J=1,NU)
0016                  IF(NUMRUN.GT.1) GO TO 150
0017                4 DO 102 I=1,NX
0018              102 READ(IIN,901)(H(I,J),J=1,NX)
0019                  IF(NUMRUN.GT.1) GO TO 150
0020                5 DO 103 I=1,NX
0021              103 READ(IIN,901)(Z(I,J),J=1,NPP)
0022                  IF(NUMRUN.GT.1) GO TO 150
                C
                C  INITIALIZE SYSTEM
                C
0023                6 READ(IIN,901)(X(I,1),I=1,NX)
0024                  IF(NUMRUN.GT.1) GO TO 150
0025                7 DO 110 I=1,NU
0026              110 READ(IIN,901)(U(I,J),J=1,NPP)
0027                  IF(NUMRUN.EQ.1) GO TO 1000
0028              150 READ(IIN,922) ICHNGE
0029                  GO TO (1000,2,3,4,5,6,7),ICHNGE
                C
                C  CALCULATE INITIAL STATE VARIABLES
                C
0030             1000 NCYCLE=0
0031                  DO 120 L=1,NPP
0032                  M=L+1
0033                  DO 121 I=1,NX
0034              121 X(I,M)=0.0
0035                  DO 120 I=1,NX
0036                  DO 122 J=1,NX
0037              122 X(I,M)=X(I,M)+F(I,J)*X(J,L)+H(I,J)*Z(J,L)
0038                  DO 120 J=1,NU
```

```
FORTRAN IV G LEVEL  18                  MAIN                DATE = 71300        10/27/59

0039          120 X(I,M)=X(I,M)+G(I,J)*U(J,L)
                C
                C  ECHO CHECK INITIAL DATA
                C
0040                WRITE(IOUT,902) NUMRUN
0041                WRITE(IOUT,903) (I,I=1,NX)
0042                DO 200 I=1,NX
0043          200 WRITE(IOUT,904) I,(F(I,J),J=1,NX)
0044                WRITE(IOUT,905)
0045                WRITE(IOUT,903) (I,I=1,NU)
0046                DO 201 I=1,NX
0047          201 WRITE(IOUT,904) I,(G(I,J),J=1,NU)
0048                WRITE(IOUT,906)
0049                WRITE(IOUT,903)(I,I=1,NX)
0050                DO 202 I=1,NX
0051          202 WRITE(IOUT,904) I,(H(I,J),J=1,NX)
0052                WRITE(IOUT,907)
0053                WRITE(IOUT,903) (I,I=1,NX)
0054                DO 203 J=1,NPP
0055          203 WRITE(IOUT,904) J,(Z(I,J),I=1,NX)
                C
                C  BEGIN ITERATIVE PROCEDURE
                C
0056                CALL CALCV(0)
0057          300 WRITE(IOUT,908) NCYCLE
0058                WRITE(IOUT,909) (I,I=1,NU)
0059                DO 301 J=1,NPP
0060          301 WRITE(IOUT,910) J,(U(I,J),I=1,NU)
0061                WRITE(IOUT,911)
0062                WRITE(IOUT,909) (J,J=1,LPP)
0063                DO 302 I=1,NX
0064          302 WRITE(IOUT,912) I,(X(I,J),J=1,LPP)
0065                CALL CALCV(LPP)
                C
                C  CALCULATE J-VALUE
                C
0066                JJ=0.0
0067                DO 310 I=1,LPP
0068          310 JJ=JJ+V(I)
0069                WRITE(IOUT,913)(I,I=1,LPP)
0070                WRITE(IOUT,914)(V(I),I=1,LPP)
0071                WRITE(IOUT,915)JJ
                C
                C  CHECK TO SEE IF CURRENT CONTROLS ARE OPTIMAL
                C
0072                CHECK=ABS(JJ-OPT)
0073                IF(CHECK.GT.0.02) GO TO 320
0074                WRITE(IOUT,916)
0075                GO TO 1
0076          320 IF(NUMIT-NCYCLE) 321,322,322
0077          321 WRITE(IOUT,917)
0078                GO TO 1
                C
                C  BEGIN MAJOR LOOP
                C
0079          322 AA=0.0
0080                DO 400 II=1,NPP
0081                IT=LPP-II
```

```
FORTRAN IV G LEVEL  18                  MAIN                DATE = 71300          10/27/59

0082                  CALL CALCV(IT)
                C
                C  CALCULATE MATRIX A = VXX + F' VXX(T+1) F
                C
0083                  DO 401 J=1,NX
0084                  DO 401 I=1,NX
0085                  A(I,J)=0.0
0086                  DO 401 K=1,NX
0087              401 A(I,J)=A(I,J)+F(K,I)*VXX2(K,J)
0088                  DO 402 J=1,NX
0089                  DO 402 I=1,NX
0090                  W(I,J)=0.0
0091                  DO 402 K=1,NX
0092              402 W(I,J)=W(I,J)+A(I,K)*F(K,J)
0093                  DO 403 J=1,NX
0094                  DO 403 I=1,NX
0095              403 A(I,J)=W(I,J)+VXX(I,J)
                C
                C  CALCULATE MATRIX B = VUX + G' VXX(T+1) F
                C
0096                  DO 404 I=1,NU
0097                  DO 404 J=1,NX
0098                  B(I,J)=0.0
0099                  DO 404 K=1,NX
0100              404 B(I,J)=B(I,J)+G(K,I)*VXX2(K,J)
0101                  DO 405 I=1,NU
0102                  DO 405 J=1,NX
0103                  W(I,J)=0.0
0104                  DO 405 K=1,NX
0105              405 W(I,J)=W(I,J)+B(I,K)*F(K,J)
0106                  DO 406 I=1,NU
0107                  DO 406 J=1,NX
0108              406 B(I,J)=W(I,J)+VUX(I,J)
                C
                C  CALCULATE MATRIX C = VUU + G' VXX(T+1) G
                C
0109                  DO 407 I=1,NU
0110                  DO 407 J=1,NX
0111                  W(I,J)=0.0
0112                  DO 407 K=1,NX
0113              407 W(I,J)=W(I,J)+G(K,I)*VXX2(K,J)
0114                  DO 408 I=1,NU
0115                  DO 408 J=1,NU
0116                  C(I,J)=0.00
0117                  DO 408 K=1,NX
0118              408 C(I,J)=C(I,J)+W(I,K)*G(K,J)
0119                  DO 409 I=1,NU
0120                  DO 409 J=1,NU
0121              409 C(I,J)=C(I,J)+VUU(I,J)
                C
                C  PLACE UPPER TRIANGLE OF C INTO CC BY COLS.
                C
0122                  K=1
0123                  DO 419 J=1,NU
0124                  DO 418 I=1,J
0125                  IF(C(I,J)) 415,416,416
0126              415 CC(K)=DMAX1(C(I,J),C(J,I))
0127                  GO TO 418
```

```
FORTRAN IV G LEVEL  18                  MAIN                DATE = 71300        10/27/59

0128          416 CC(K)=DMIN1(C(I,J),C(J,I))
0129          418 K=K+1
0130          419 CONTINUE
0131              IER=0
0132              EPS=.000001
0133              CALL DSINV(CC,NU,EPS,IER)
                  C
                  C  PLACE CC IN CINV
                  C
0134              L=1
0135              DO 485 J=1,NU
0136              DO 480 I=1,J
0137              CINV(I,J)=CC(L)
0138              IF(I.NE.J) CINV(J,I)=CC(L)
0139          480 L=L+1
0140          485 CONTINUE
0141              IF(IER.EQ.-1) GO TO 1
0142              DO 410 J=1,NU
0143              HU(J)=0.0
0144              DO 411 I=1,NX
0145          411 HU(J)=HU(J)+VX2(I)*G(I,J)
0146          410 HU(J)=HU(J)+VU(J)
                  C
                  C  CALCULATE ALPHA(IT),BETA(IT)
                  C
0147              DO 420 I=1,NU
0148              ALPHA(I,IT)=0.0
0149              DO 421 J=1,NU
0150          421 ALPHA(I,IT)=ALPHA(I,IT)+CINV(I,J)*HU(J)
0151          420 ALPHA(I,IT)=-ALPHA(I,IT)
0152              DO 430 I=1,NU
0153              DO 430 J=1,NX
0154              BETA(I,J,IT)=0.0
0155              DO 431 K=1,NU
0156          431 BETA(I,J,IT)=BETA(I,J,IT)+CINV(I,K)*B(K,J)
0157          430 BETA(I,J,IT)=-BETA(I,J,IT)
                  C
                  C  CALCULATE NEW VXX2 AND NEW VX2
                  C
0158              DO 440 I=1,NX
0159              W(I,1)=0.0
0160              DO 440 J=1,NU
0161          440 W(I,1)=W(I,1)+BETA(J,I,IT)*HU(J)
0162              DO 441 I=1,NX
0163              W(I,2)=0.0
0164              DO 441 J=1,NX
0165          441 W(I,2)=W(I,2)+VX2(J)*F(J,I)
0166              DO 442 I=1,NX
0167          442 VX2(I)=VX(I)+W(I,1)+W(I,2)
0168              DO 450 I=1,NU
0169              DO 450 J=1,NX
0170              W(I,J)=0.0
0171              DO 450 K=1,NU
0172          450 W(I,J)=W(I,J)+CINV(I,K)*B(K,J)
0173              DO 451 I=1,NX
0174              DO 451 J=1,NX
0175              VXX2(I,J)=0.0
0176              DO 452 K=1,NU
```

```
FORTRAN IV G LEVEL  18                  MAIN                    DATE = 71300          10/27/59

0177            452 VXX2(I,J)=VXX2(I,J)+B(K,I)*W(K,J)
0178            451 VXX2(I,J)=A(I,J)-VXX2(I,J)
                C
                C  CALCULATE AA TO INDICATE EXPECTED IMPROVEMENT IN J-VALUE
                C
0179                W(1,1)=0.0
0180                DO 460 I=1,NU
0181            460 W(1,1)=W(1,1)+HU(I)*ALPHA(I,IT)
0182            490 AA=AA+0.5*W(1,1)
                C
                C  CALCULATE NEW CONTROLS AND NEW STATE VARIABLES
                C
0183                DO 500 I=1,NU
0184                U(I,1)=U(I,1)+ALPHA(I,1)
0185                IF(U(I,1).LT.0.) U(I,1)=0.
0186            500 CONTINUE
0187                DO 501 I=1,NX
0188                XNEW(I,2)=0.0
0189                DO 502 J=1,NX
0190            502 XNEW(I,2)=XNEW(I,2)+F(I,J)*X(J,1)+H(I,J)*Z(J,1)
0191                DO 501 J=1,NU
0192            501 XNEW(I,2)=XNEW(I,2)+G(I,J)*U(J,1)
0193                DO 510 L=2,NPP
0194                M=L+1
0195                DO 511 I=1,NU
0196                DELU(I)=0.0
0197                DO 512 J=1,NX
0198            512 DELU(I) =     BETA(I,J,L)*(XNEW(J,L)-X(J,L))+DELU(I)
0199                U(I,L)= DELU(I)+ALPHA(I,L)+U(I,L)
0200                IF(U(I,L).LT.0.) U(I,L)=0.
0201            511 CONTINUE
0202                DO 513 I=1,NX
0203                XNEW(I,M)=0.0
0204                DO 514 J=1,NX
0205            514 XNEW(I,M)=XNEW(I,M)+F(I,J)*XNEW(J,L)+H(I,J)*Z(J,L)
0206                DO 513 J=1,NU
0207            513 XNEW(I,M)=XNEW(I,M)+G(I,J)*U(J,L)
0208            510 CONTINUE
0209                DO 520 I=1,NX
0210                DO 520 J=2,LPP
0211            520 X(I,J)=XNEW(I,J)
0212                NCYCLE=NCYCLE+1
0213                WRITE(IOUT,918)AA
0214                GO TO 300
0215            900 FORMAT(6I3,F8.0)
0216            901 FORMAT(10F8.0)
0217            902 FORMAT(1H1,20X,'DATA FOR RUN NUMBER',I3//20X,'F-MATRIX')
0218            903 FORMAT(/3X,14I9)
0219            904 FORMAT(1X,I2,14F9.2)
0220            905 FORMAT(//20X,'G-MATRIX')
0221            906 FORMAT(1H1,20X,'H-MATRIX')
0222            907 FORMAT(//20X,'Z-TRANSPOSE')
0223            908 FORMAT(1H1,10X,'STATISTICS FOR CYCLE NUMBER',I4//20X,'CONTROL VARI
                   1ABLES')
0224            909 FORMAT(/(3X,10I12))
0225            910 FORMAT(1X,I2,5X,10G12.4)
0226            911 FORMAT(1H1,20X,'STATE VARIABLES')
0227            912 FORMAT(/2X,'X(',I2,') ',10G12.4/(8X,10G12.4/))
```

```
FORTRAN IV G LEVEL  18                          DATE = 71300          10/27/59
C ***********************************************************************
C
C    PURPOSE
C     INVERT A GIVEN SYMMETRIC POSITIVE DEFINITE MATRIX
C
C    USAGE
C     CALL DSINV(A,N,EPS,IER)
C
C    DESCRIPTION OF PARAMETERS
C     A    DOUBLE PRECISION UPPER TRIANGULAR PART OF GIVEN SYMMETRIC
C          POSITIVE DEFINITE N BY N MATRIX.
C          ON RETURN A CONTAINS THE RESULTANT UPPERTRIANGULAR MATRIX
C          IN DOUBLE PRECISION.
C     N    ORDER OF THE GIVEN MATRIX
C     EPS  SINGLE PRECISION INPUT PARAMETER WHICH IS USED AS RELATIVE
C          TOLERANCE FOR TEST ON LOSS OF SIGNIFICANCE.
C     IER  RESULTING ERROR PARAMETER CODED AS FOLLOWS
C          IER=0     NO ERROR
C          IER=-1    NO RESULT BECAUSE OF WRONG INPUT PARAMETER N OR
C                    BECAUSE SOME RADICAND IS NON-POSITIVE (MATRIX A IS
C                    IS NOT POSITIVE DEFINITE, POSSIBLY DUE TO LOSS OF
C                    SIGNIFICANCE,)
C          IER=5     WARNING WHICH INDICATES LOSS OF SIGNIFICANCE.
C
C    REMARKS
C     THE UPPER TRIANGULAR PART OF GIVEN MATRIX IS ASSUMED TO BE STORED
C     COLUMNWISE IN N*(N+1)/2 SUCCESSIVE STORAGE LOCATIONS.  IN THE
C     SAME STORAGE LOCATIONS THE RESULTING UPPER TRIANGULAR MATRIX IS
C     STORED COLUMNWISE TOO.
C     THE PROCEDURE GIVES RESULTS IF N IS GREATER THAN 0 AND ALL
C     CALCULATED RADICANDS ARE POSITIVE.
C
C    SUBROUTINE REQUIRED - DMFSD
C
C ***********************************************************************
0001            SUBROUTINE DSINV(A,N,EPS,IER)
C
0002            DIMENSION A(1)
0003            DOUBLE PRECISION A,DIN,WORK
C
C               FACTORIZE GIVEN MATRIX BY MEANS OF SUBROUTINE DMFSD
C               A=TRANSPOSE(T) * T
0004            CALL DMFSD(A,N,EPS,IER)
0005            IF(IER) 9,1,1
C
C               INVERT UPPER TRIANGULAR MATRIX T
C               PREPARE INVERSION-LOOP
0006          1 IPIV=N*(N+1)/2
0007            IND=IPIV
C
C               INITIALIZE INVERSION-LOOP
0008            DO 6 I=1,N
0009            DIN=1.D0/A(IPIV)
0010            A(IPIV)=DIN
0011            MIN=N
0012            KEND=I-1
0013            LANF=N-KEND
0014            IF(KEND)5,5,2
```

```
FORTRAN IV G LEVEL  18                  DSINV                DATE = 71300        10/27/59

0015              2 J=IND
                C
                C         INITIALIZE ROW-LOOP
0016                DO 4 K=1,KEND
0017                WORK=0.D0
0018                MIN=MIN-1
0019                LHOR=IPIV
0020                LVER=J
                C
                C         START INNER LOOP
0021                DO 3 L=LANF,MIN
0022                LVER=LVER+1
0023                LHOR=LHOR+L
0024              3 WORK=WORK+A(LVER)*A(LHOR)
                C         END OL INNER LOOP
                C
0025                A(J)=-WORK*DIN
0026              4 J=J-MIN
                C         END OF ROW-LOOP
                C
0027              5 IPIV=IPIV-MIN
0028              6 IND=IND-1
                C         END OF INVERSION-LOOP
                C
                C         CALCULATE INVERSE(A) BY MEANS OF INVERSE(T)
                C         INVERSE(A)=INVERSE(T)*TRANSPOSE(INVERSE(T))
                C         INITIALIZE MULTIPLICATION-LOOP
0029                DO 8 I=1,N
0030                IPIV=IPIV+I
0031                J=IPIV
                C
                C         INITIALIZE ROW-LOOP
0032                DO 8 K=I,N
0033                WORK=0.D0
0034                LHOR=J
                C
                C         START INNER LOOP
0035                DO 7 L=K,N
0036                LVER=LHOR+K-I
0037                WORK=WORK+A(LHOR)*A(LVER)
0038              7 LHOR=LHOR+L
                C         END OF INNER LOOP
                C
0039                A(J)=WORK
0040              8 J=J+K
                C         END OR ROW- AND MULTIPLICATION-LOOP
                C
0041              9 RETURN
0042                END
```

```
C ************************************************************************
C
C    PURPOSE
C     FACTOR A GIVEN SYMMETRIC POSITIVE DEFINITE MATRIX
C
C    USAGE
C     CALL DMFSD(A,N,EPS,IER)
C
C    DESCRIPTION OF PARAMETERS
C     A     DOUBLE PRECISION UPPER TRIANGULAR PART OF GIVEN
C           SYMMETRIC POSITIVE DEFINITE N BY N MATRIX
C     N     THE NUMBER OF ROWS(COLUMNS) IN GIVEN MATRIX
C           ON RETURN A CONTAINS THE RESULTANT UPPER
C           TRIANGULAR MATRIX IN DOUBLE PRECISION
C     EPS   SINGLE PRECISION INPUT CONSTANT WHICH IS USED
C           AS RELATIVE TOLERANCE FOR TEST ON LOSS OF
C           SIGNIFICANCE
C     IER   RESULTING ERRORPARAMETER CODED AS FOLLOWS
C           IER=0 - NO ERROR
C           IER=-1 - NO RESULT BECAUSE OF WRONG INPUT PARAMETER
C                    N OR BECAUSE SOME RADICAND IS NON-POSITIVE
C                    (MATRIX A IS NOT POSITIVEDEFINITE -
C                    POSSIBLY DUE TO LOSS OF SIGNIFICANCE)
C           IER=K  - WARNING WHICH INDICATES LOSS OF SIGNIFICANCE
C                    THE RADICAND FORMED AT FACTORIZATION
C                    STEP K+1 WAS STILL POSITIVE BUT NO LONGER
C                    GREATER THAN ABS(EPS*K+1,K+1))
C    REMARKS
C     THE UPPER TRIANGULAR PART OF GIVEN MATRIX IS ASSUMED TO BE
C     STORED COLUMNWISE IN N*(N+1)/2 SUCCESSIVE STORAGE LOCATIONS.
C     IN THE SAME STORAGE LOCATIONS THE RESULTING UPPER TRIANGULAR
C     MATRIX IS STORED COLUMNWISE TOO.
C     THE PROCEDURE GIVES RESULTS IF N IS GREATER THAN 0 AND ALL
C     CALCULATED RADICANDS ARE POSITIVE.
C     THE PRODUCT OF RETURNED DIAGONAL TERMS IS EQUAL TO THE
C     SQUARE-ROOT OF THE DETERMINANT OF THE GIVEN MATRIX.
C
C    METHOD
C     SOLUTION IS DONE USING THE SQUARE-ROOT METHOD OF CHOLESKY.
C     THE GIVEN MATRIX IS REPRESENTED AS PRODUCT OF TWO TRIANGULAR
C     MATRICES. THE LEFT HAND FACTOR IS THE TRANSPOSE OF THE
C     THE RETURNED RIGHT HAND FACTOR.
C
C ************************************************************************
0001          SUBROUTINE DMFSD(A,N,EPS,IER)
C
C
0002          DIMENSION A(1)
0003          DOUBLE PRECISION DPIV,DSUM,A
C
C        TEST ON WRONG INPUT PARAMETER N
0004          IF(N-1)12,1,1
0005        1 IER=0
C
C        INITIALIZE DIAGNONAL-LOOP
0006          KPIV=0
0007          DO 11 K=1,N
0008          KPIV=KPIV+K
```

```
FORTRAN IV G LEVEL  18                  DMFSD                DATE = 71300          10/27/59

0009                      IND=KPIV
0010                      LEND=K-1
                  C
                  C          CALCULATE TOLERANCE
0011                      TOL=ABS(EPS*SNGL(A(KPIV)))
                  C
                  C          START FACTORIZATION-LOOP OVER K-TH ROW
0012                      DO 11 I=K,N
0013                      DSUM=0.D0
0014                      IF(LEND)2,4,2
                  C
                  C          START INNER LOOP
0015                    2 DO 3 L=1,LEND
0016                      LANF=KPIV-L
0017                      LIND=IND-L
0018                    3 DSUM=DSUM+A(LANF)*A(LIND)
                  C          END OF INNER LOOP
                  C
                  C          TRANSFORM ELEMENT A(IND)
0019                    4 DSUM=A(IND)-DSUM
0020                      IF(I-K) 10,5,10
                  C
                  C          TEST FOR NEGATIVE PIVOT ELEMENT AND FOR LOSS OF SIGNIFICANCE
0021                    5 IF(SNGL(DSUM)-TOL) 6,6,9
0022                    6 IF(DSUM) 12,12,7
0023                    7 IF(IER) 8,8,9
0024                    8 IER=K-1
                  C
                  C          COMPUTE PIVOT ELEMENT
0025                    9 DPIV=DSQRT(DSUM)
0026                      A(KPIV)=DPIV
0027                      DPIV=1.D0/DPIV
0028                      GO TO 11
                  C
                  C          CALCULATE TERMS IN ROW
0029                   10 A(IND)=DSUM*DPIV
0030                   11 IND=IND+I
                  C          END OF DIAGONAL LOOP
                  C
0031                      RETURN
0032                   12 IER=-1
0033                      RETURN
0034                      END
```

```
FORTRAN IV G LEVEL  18                  CALCV                DATE = 71300          10/27/59

0001                  SUBROUTINE CALCV(IPER)
                C  IN WHICH IS CALCULATED VX,VU,VXX,VUX,VUU,AND V
                C     THERE ARE THREE SECTIONS TO THE ROUTINE
                C    1 PARAMETERS ARE READ IN,SECOND DERIVATIVE MATRICES SET TO 0.0
                C    2 V IS CALCULATED USING UPDATED X AND U,ALSO VX2(LPP) AND UXX2(LPP)
                C     SUMMARY STATISTICS ARE PRINTED OUT
                C    3 VX,VU,VXX,VUX,VUU, ARE CALCULATED FOR PERIOD IT AS DEFINED IN MAIN
                C
0002                  COMMON X(14,20),U(09,19),NX,NU,NPP,VX(14),   VXX(14,14),VUX(9,14),
                     1VUU(9,9),VXX2(14,14),VU(9),LPP,VX2(14),V(20),F(14,14),G(14,9),
                     2IIN,IOUT,NUMRUN
0003                  DIMENSION R(8),P(8)
0004                  REAL K(9)
0005                  IF(IPER.EQ.0) GO TO 100
0006                  IF(IPER.EQ.LPP) GO TO 200
0007                  GO TO 300
                C
                C  READ BASIC DATA
                C
0008                100 IF(NUMRUN.EQ.1) GO TO 103
0009                  READ(IIN,904) ICHNGE
0010                  IF(ICHNGE.EQ.0) GO TO 101
0011                103 READ(IIN,900) K,R,C1,C2,C3
0012                101 WRITE(IOUT,901) K,R,C1,C2,C3
                C
                C CALCULATE VXX,VUX,VUU (WHICH IS INVARIANT)
                C
0013                  DO 102 J=1,14
0014                  DO 102 I=1,J
0015                102 VXX(I,J)=0.0
0016                  D=K(1)*R(1)**2 +K(5)*C3**2
0017                  VXX(1,1)=D+K(2)*R(2)**2+K(3)*R(3)**2+K(4)*R(4)**2
0018                  VXX(1,2)=D-K(2)*R(2)
0019                  VXX(1,3)=D-K(3)*R(3)
0020                  VXX(1,4)=D-K(4)*R(4)
0021                  VXX(2,2)=D+K(2)
0022                  VXX(2,3)=D
0023                  VXX(2,4)=D
0024                  VXX(3,3)=D+K(3)
0025                  VXX(3,4)=D
0026                  VXX(4,4)=D+K(4)
0027                  DO 110 I=1,4
0028                  DO 110 J=5,8
0029                110 VXX(I,J)=-K(1)*R(1)+K(5)*C2*C3
0030                  DO 111 I=1,4
0031                111 VXX(I,12) = -K(5)*C3
0032                  D=K(1)+K(5)*C2**2
0033                  VXX(5,5)=D+K(6)*R(6)**2+K(7)*R(7)**2+K(8)*R(8)**2
0034                  VXX(5,6)=D-K(6)*R(6)
0035                  VXX(5,7)=D-K(7)*R(7)
0036                  VXX(5,8)=D-K(8)*R(8)
0037                  VXX(6,6)=D+K(6)
0038                  VXX(6,7)=D
0039                  VXX(6,8)=D
0040                  VXX(7,7)=D+K(7)
0041                  VXX(7,8)=D
0042                  VXX(8,8)=D+K(8)
0043                  DO 112 I=5,8
```

```
FORTRAN IV G LEVEL  18                  CALCV                DATE = 71300          10/27/59

0044          112 VXX(I,12)=-K(5)*C2
0045              VXX(12,12)=K(5)
0046              DO 121 J=1,14
0047              DO 121 I=1,J
0048          121 VXX(I,J)=K(9)*F(14,I)*F(14,J) +VXX(I,J)
0049              DO 120 I=2,14
0050              IM1=I-1
0051              DO 120 J=1,IM1
0052          120 VXX(I,J)=VXX(J,I)
0053              DO 130 I=1,8
0054              DO 130 J=1,8
0055              VUU(I,J)=VXX(I,J)
0056          130 VUX(I,J)=VXX(I,J)
0057              DO 131 I=1,8
0058              DO 131 J=9,14
0059          131 VUX(I,J)=VXX(I,J)
0060              DO 132 J=1,14
0061          132 VUX(9,J)=K(9)*F(14,J)*G(14,9)
0062              DO 133 J=1,8
0063              VUU(9,J)=VUX(9,J)
0064          133 VUU(J,9)=VUU(9,J)
0065              VUU(9,9)=K(9)*G(14,9)**2
0066              GO TO 800
                C
                C  CALCULATE PREFERENCE FUNCTION AND VXX2(LPP),VX2(LPP)
                C
0067          200 WRITE(IOUT,902)
0068              DO 210 J=1,NPP
0069              DO 201 I=1,8
0070          201 P(I)=X(I,J)+U(I,J)
0071              TS=0.0
0072              TF=0.0
0073              DO 202 I=1,4
0074              TF=TF+P(I)
0075          202 TS=TS+P(I+4)
0076              P(2)=P(2)/P(1)
0077              P(3)=P(3)/P(1)
0078              P(4)=P(4)/P(1)
0079              P(1)=TS/TF
0080              A1=P(6)/P(5)
0081              A2=P(7)/P(5)
0082              A3=P(8)/P(5)
0083              A4=C1+C2*TS+C3*TF
0084              WRITE(IOUT,903)J,TS,TF,(P(I),I=1,4),A1,A2,A3,A4,X(12,J)
0085              A1=R(1)*TF-TS
0086              A2=R(2)*(X(1,J)+U(1,J))-(X(2,J)+U(2,J))
0087              A3=R(3)*(X(1,J)+U(1,J))-(X(3,J)+U(3,J))
0088              A4=R(4)*(X(1,J)+U(1,J))-(X(4,J)+U(4,J))
0089              A5=C1+C2*TS+C3*TF-X(12,J)
0090              A6=R(6)*(X(5,J)+U(5,J))-(X(6,J)+U(6,J))
0091              A7=R(7)*(X(5,J)+U(5,J))-(X(7,J)+U(7,J))
0092              A8=R(8)*(X(5,J)+U(5,J))-(X(8,J)+U(8,J))
0093              V(J)=K(1)*A1**2+K(2)*A2**2+K(3)*A3**2+K(4)*A4**2+K(5)*A5**2
                 1+K(6)*A6**2+K(7)*A7**2+K(8)*A8**2+K(9)*X(14,J+1)**2
0094          210 V(J)=V(J)/2.0
0095              V(LPP)=0.5*K(9)*X(14,LPP)**2
                 1 + 0.5*R(5)*(X(9,LPP)**2+X(10,LPP)**2+X(11,LPP)**2)
0096              DO 220 I=1,13
```

```
FORTRAN IV G LEVEL  18                  CALCV                DATE = 71300          10/27/59

0097            220 VX2(I)=0.0
0098                DO 221 I=1,14
0099                DO 221 J=1,14
0100            221 VXX2(I,J)=0.0
0101                DO 222 I=9,11
0102                VX2(I)=R(5)*X(I,LPP)
0103            222 VXX2(I,I)=R(5)
0104                VX2(14)=    K(9)*X(14,LPP)
0105                VXX2(14,14)=K(9)
0106                GO TO 800
                C
                C  CALCULATE VX(IPER)
                C
0107            300 TF=0.0
0108                DO 301 I=1,4
0109            301 TF=TF+X(I,IPER)+U(I,IPER)
0110                LD = X(5,IPER) + X(6,IPER) +U(5,IPER)+U(6,IPER)
0111                UD = X(7,IPER) + X(8,IPER) +U(7,IPER)+U(8,IPER)
0112                TS = LD + UD
0113                DO 302 I=1,14
0114            302 VX(I)=0.0
0115                J=IPER
0116                A1=R(1)*TF-TS
0117                A2=R(2)*(X(1,J)+U(1,J))-(X(2,J)+U(2,J))
0118                A3=R(3)*(X(1,J)+U(1,J))-(X(3,J)+U(3,J))
0119                A4=R(4)*(X(1,J)+U(1,J))-(X(4,J)+U(4,J))
0120                A5=C1+C2*TS+C3*TF-X(12,J)
0121                A6=R(6)*(X(5,J)+U(5,J))-(X(6,J)+U(6,J))
0122                A7=R(7)*(X(5,J)+U(5,J))-(X(7,J)+U(7,J))
0123                A8=R(8)*(X(5,J)+U(5,J))-(X(8,J)+U(8,J))
0124                D=K(1)*A1*R(1)+K(5)*A5*C3
0125                VX(1)=D+K(2)*A2*R(2)+K(3)*A3*R(3)+K(4)*A4*R(4)
0126                VX(2)=D-K(2)*A2
0127                VX(3)=D-K(3)*A3
0128                VX(4)=D-K(4)*A4
0129                D=-K(1)*A1+K(5)*A5*C2
0130                VX(5)=D+K(6)*A6*R(6)+K(7)*A7*R(7)+K(8)*A8*R(8)
0131                VX(6)=D-K(6)*A6
0132                VX(7)=D-K(7)*A7
0133                VX(8)=D-K(8)*A8
0134                VX(12)=-K(5)*A5
0135                DO 303 I=1,14
0136            303 VX(I)=VX(I)+K(9)*X(14,J+1)*F(14,I)
0137                DO 310 I=1,8
0138            310 VU(I)=VX(I)
0139                VU(9)=K(9)*X(14,J+1)*G(14,9)
0140            800 CONTINUE
0141                RETURN
                C
0142            900 FORMAT(9F8.0/8F8.0/3F8.0)
0143            901 FORMAT(1H1,20X'PREFERENCE FUNCTION WEIGHTS'//1X'K1-K9',9F12.6//
                   11X'R1-R8',8F12.6//1X,'C1-C3',3F12.6)
0144            902 FORMAT(1H1,50X,'SUMMARY STATISTICS'//1X,'PER',4X,'TS',9X,'TF',8X,
                   1'TS/TF',6X,'F2/F1',6X,'F3/F1',6X,'F4/F1',6X,'SO/FR',6X,'JR/FR',
                   26X,'SR/FR',5X,'ASF REQ',6X,'ASF BUILT'/)
0145            903 FORMAT(1X,I3,2(3X,F8.1),7(3X,F8.3),2(3X,F9.2))
0146            904 FORMAT(I1)
0147                END
```

## DATA INPUTS FOR MULTIPLE RUNS

To make multiple comparison runs without respecifying all the data inputs, coded data-change cards are used. Changes may be made in either the data inputs to the main routine or to CALCV or both.

Changes in the data for MAIN must be preceded by a RUN card and followed by a card with "1" in column 1. Changes made in CALCV data must be preceded by a card with "2" in column 1. If no changes are made in MAIN data, a card with "1" in column 1 must follow the RUN card. If no changes are made in CALCV data, a card with "Ø" (zero) must follow the last card in the data set which changes MAIN, i.e., the card with "1" in column 1. The END card which follows the data in single runs is to be made the last card of the change-data sets. (There is only one END card for the program.)

# FORM OF INPUT DATA

## I. DATA FOR MAIN ROUTINE

| Item | Var.Name | Value | # Cols. | # Rows | Format |
|---|---|---|---|---|---|
| 1) RUN card: | | | | | (I3,3x,4I3,F8.0) |
| a) # Iterations this run | NUMIT | | | | |
| b) # Control Variables | NU | | | | |
| c) # State Variables | NX | | | | |
| d) # Planning Periods | NPP | | | | |
| e) Run Number | NUMRUN | 001 | | | |
| f) Est. Optimal Value of p.f. | OPT | | | | |
| 2) State Variable Transition Matrix (read in by rows) | F | | NX | NX | (10 F8.0) |
| 3) Control Variable Transition Matrix (read in by rows) | G | | NX | NU | (10 F8.0) |
| 4) Exogenous Variable Transition Matrix (read in by rows) | H | | NX | NX | (10 F8.0) |
| 5) Exogenous Variables for all Periods (read in by rows) | Z | | NPP | NX | (10 F8.0) |
| 6) Initial State Variables | X | | | 1 | (10 F8.0) |
| 7) Initial Control Variables for all Periods (read in by rows) | U | | NPP | NU | (10 F8.0) |

## II. DATA FOR CALCV SUBROUTINE

| | | | | | |
|---|---|---|---|---|---|
| 1) Vector of p.f. Weights | K | | 9 | 1 | (9 F8.0) |
| 2) Vector of p.f. Ratios (Note R(5) is wt for LPP $x_9, x_{10}, x_{11}$) | R | | 8 | 1 | (8 F8.0) |
| 3) Vector of Coefficients for ASF Needs | C | | 3 | 1 | (3 F8.0) |
| END CARD | | 100 | | | (I3) |

## DATA STRUCTURE FOR MULTIPLE RUNS

| Item | Var.Name | Value | # Cols. | # Rows | Format |
|---|---|---|---|---|---|
| 1) RUN card (as before)<br>2) MAIN data (as before)<br>3) CALCV data (as before) | | | | | |
| 4) RUN card (as before except run number)<br>e) | NUMRUN | >002 | | | |
| 5) Change code<br>New F-Matrix follows<br>New G-Matrix follows<br>New H-Matrix follows<br>New Z-Matrix follows<br>New Initial-state Variables follow<br>New Initial-control Variables follow<br>No (further) changes in MAIN data<br>Following each change coded card, the appropriate variables are specified as before. | ICHNGE | 1-7<br>2<br>3<br>4<br>5<br>6<br>7<br>1 | | | (I1) |
| 6a) If there are no changes in CALCV data | ICHNGE | 0 | | | (I1) |
| 6b) If there are changes in CALCV data<br>Complete set of CALCV data as before | ICHNGE | 2 | | | (I1) |
| [Repeat 4 - 6 as needed] | | | | | |
| 7) END CARD | | 100 | | | (I3) |

# BIBLIOGRAPHY

[1] Aoki, M., Optimization of Stochastic Systems, Academic Press, 1967.

[2] Balderston, Frederick E., "The Varities of the Financial Crisis," Universal Higher Education, Costs and Benefits, American Council on Education, Washington, D.C., ACE Conference, October 6-8, 1971.

[3] Baybrooke, David and C. E. Lindblom, A Strategy of Decision, Macmillan Co., 1963.

[4] Breneman, David and George Weathersby, "Definition and Measurement of the Activites and Outputs of Higher Education," Ford Foundation Program in University Administration, Discussion Paper No. 10, University of California, Berkeley, 1970.

[5] Brode, Wallace R., "Manpower in Science and Engineering, Based on a Saturation Model," Science, Vol. 173, July 16, 1971.

[6] Cheit, Earl C., The New Depression in Higher Education, McGraw Hill Co., 1970.

[7] Downs, Anthony, The Economics of Democracy, Harper & Row, 1967.

[8] Geoffrion, Arthur M., "Vector Maximal Decomposition Programming," Working Paper No. 164, Western Management Science Institute, UCLA, 1970.

[9] _____, and James Dyer, "Academic Departmental Management: An Application of an Interactive Multi-criterion Optimization Approach," Ford Foundation Program in University Administration, Paper P-25, University of California, Berkeley, 1971.

[10] Glenny, L. A., Medsker, L. L., Palola. E. G., and J. G. Paltridge, "A Survey of Research and Perspectives on National Planning for Higher Education," Berkeley: Center for Research and Development in Higher Education, 1969.

[11] Huff, Robert A., Inventory of Educational Outcomes and Activities, Technical Report 15, Western Interstate Commission on Higher Education, Boulder, Colorado, 1971.

[12] Jacobson, David H., "New Second-Order and First-Order Algorithms for Determining Optimal Control: A Differential Dynamic Programming Approach," Journal of Optimal Theory and Applications, Vol. 2, No. 6, 1968.

[13] Jewett, James E., "College Admissions Planning: Use of a Student Segmentation Model," Ford Foundation Program in University Administration Paper P-23, University of California, Berkeley, 1971.

[14] Lawrence, Ben, George Weathersby and Virginia Patterson, The Outputs of Higher Education, Western Interstate Commission on Higher Education, Boulder, Colorado, 1970.

[15] Mayne, David, "A Second Order Gradient Method for Determining Optimal Trajectories of Non-Linear Discrete-Time Systems," International Journal of Control, Vol. 3, No. 1, 1966.

[16] Miller, Leonard S., "Demand for Higher Education in the United States," National Bureau of Economic Research, 1971.

[17] O'Neill, June, Resource Use in Higher Education, Carnegie Commission on Higher Education, 1971.

[18] Palola, E.G., T. Lehmann, and W. R. Blischke, Higher Education by Design: The Sociology of Planning, Berkeley: Center for Research and Development in Higher Education, 1970.

[19] Pratt, John W., "Risk Aversion in the Small and in the Large," Econometrica, Vol. 32, 1964.

[20] Weathersby, George, "Decision Analysis for University and Other Public Administrators," Ford Foundation Program in University Administration, Discussion Paper No. 2, University of California, Berkeley, 1969a.

[21] _____, "Preference Structures, Group Decision Making, and Linear Systems in Public Sector Decision Analysis," Ford Foundation Program in University Administration, Discussion Paper No. 3, University of California, Berkeley, 1969b.

[22] _____, "Student Tuition Models in Private and Public Higher Education," 38th ORSA Meeting, Detroit, Michigan, October 28-30, 1970.

[23] _____, and M.C. Weinstein, "A Structural Comparision of Analytical Models for University Planning," Ford Foundation Program in University Administration, Paper P-12, University of California, Berkeley, 1970.

[24] Wildavsky, A., The Politics of the Budgetary Process, Little Brown, Co., 1964.

## PUBLISHED REPORTS

68–3 Oliver, R. M., *Models for Predicting Gross Enrollments at the University of California.*

69–1 Marshall, K., and R. M. Oliver, *A Constant Work Model for Student Attendance and Enrollment.*

69–4 Breneman, D. W., *The Stability of Faculty Input Coefficients in Linear Workload Models of the University of California.*

69–10 Oliver, R. M., *An Equilibrium Model of Faculty Appointments, Promotions, and Quota Restrictions.*

P–1 Leimkuhler, F., and M. Cooper, *Analytical Planning for University Libraries.*

P–2 Leimkuhler, F., and M. Cooper, *Cost Accounting and Analysis for University Libraries.*

P–3 Sanderson, R. D., *The Expansion of University Facilities to Accommodate Increasing Enrollments.*

P–4 Bartholomew, D. J., *A Mathematical Analysis of Structural Control in a Graded Manpower System.*

P–5 Balderston, F. E., *Thinking About the Outputs of Higher Education.*

P–6 Weathersby, G. B., *Educational Planning and Decision Making: The Use of Decision and Control Analysis.*

P–7 Keller, J. E., *Higher Education Objectives: Measures of Performance and Effectiveness.*

P–8 Breneman, D. W., *An Economic Theory of Ph.D. Production.*

P–9 Winslow, F. D., *The Capital Costs of a University.*

P–10 Halpern, J., *Bounds for New Faculty Positions in a Budget Plan.*

P–11 Rowe, S., W. G. Wagner, and G. B. Weathersby, *A Control Theory Solution to Optimal Faculty Staffing.*

P–12 Weathersby, G. B., and M. C. Weinstein, *A Structural Comparison of Analytical Models.*

P–13 Pugliaresi, L. S., *Inquiries into a New Degree: The Candidate in Philosophy.*

P–14 Adams, R. F., and J. B. Michaelsen, *Assessing the Benefits of Collegiate Structure: The Case at Santa Cruz.*

P–15 Balderston, F. E., *The Repayment Period for Loan-Financed College Education.*

P–16 Breneman, D. W., *The Ph.D. Production Function: The Case at Berkeley.*

P–17 Breneman, D. W., *The Ph.D. Degree at Berkeley: Interviews, Placement, and Recommendations.*

P–18 Llubia, L., *An Analysis of the Schools of Business Administration at the University of California, Berkeley.*

P–19 Wing, P., *Costs of Medical Education.*

P–20 Kreplin, H. S., *Credit by Examination: A Review and Analysis of the Literature.*

P–21 Perl, L. J., *Graduation, Graduate School Attendance, and Investments in College Training.*

P–22 Wagner, W. G., and G. B. Weathersby, *Optimality in College Planning: A Control Theoretic Approach.*

P–23 Jewett, J. E., *College Admissions Planning: Use of a Student Segmentation Model.*

P–24 Breneman, D. W., (Editor), *Internal Pricing within the University—A Conference Report.*

P–25 Geoffrion, A. M., Dyer, J. S., and A. Feinberg, *Academic Departmental Management: An Application of an Interactive Multi-criterion Optimization Approach.*

P–26 Balderston, F. E., and R. Radner, *Academic Demand for New Ph.D.'s, 1970–90: Its Sensitivity to Alternative Policies.*

P–27 Morris, J., *Educational Training and Careers of Ph.D. Holders: An Exploratory Empirical Study.*

P–28 Wing, P., *Planning and Decision Making for Medical Education: An Analysis of Costs and Benefits.*

P–29 Balderston, F. E., *Varieties of Financial Crisis.*

P–30 Weathersby, G. B., *Structural Issues in the Supply and Demand for Scientific Manpower: Implications for National Manpower Policy.*

P–31 Balderston, F. E., and G. B. Weathersby, *PPBS in Higher Education Planning and Management from PPBS to Policy Analysis.*

P–32 Balderston, F. E., *Financing Postsecondary Education Statement to the Joint Committee on the Master Plan for Higher Education of the California Legislature, April 12, 1972.*

P–33 Balderston, F. E., *Cost Analysis in Higher Education.*

P–34 Smith, D. E., and W. G. Wagner, *SPACE: Space Planning and Cost Estimating Model for Higher Education.*

Single copies are available upon request; multiple copies available at cost. A price list may be obtained from the Ford Foundation office at: 2288 Fulton Street, Berkeley, California 94720.